Svör

Svör við verkefnum

Verkefni 1

1. a) $\begin{pmatrix}3\\4\end{pmatrix}$ b) $\begin{pmatrix}3\\-4\end{pmatrix}$ c) $\begin{pmatrix}12\\5\end{pmatrix}$
2. a) $B = (8,19)$ b) $B = (-16,-7)$
   c) $B = (-4,12)$ d) $B = (-8,12)$
3. a) $A = (16,7)$ b) $A = (-8,-19)$
   c) $A = (4,0)$ d) $A = (0,0)$
4. a) $\boldsymbol{a}+\boldsymbol{b} = \begin{pmatrix}3\\3\end{pmatrix}$, $\boldsymbol{a}-\boldsymbol{b} = \begin{pmatrix}-1\\-1\end{pmatrix}$
   b) $\boldsymbol{a}+\boldsymbol{b} = \begin{pmatrix}-2\\9\end{pmatrix}$, $\boldsymbol{a}-\boldsymbol{b} = \begin{pmatrix}-2\\-7\end{pmatrix}$
   c) $\boldsymbol{a}+\boldsymbol{b} = \begin{pmatrix}2\\-2\end{pmatrix}$, $\boldsymbol{a}-\boldsymbol{b} = \begin{pmatrix}-26\\26\end{pmatrix}$
5. a) $\begin{pmatrix}-5\\-8\end{pmatrix}$ b) $\begin{pmatrix}0\\0\end{pmatrix}$ c) $\begin{pmatrix}-14\\-6\end{pmatrix}$
   d) $\begin{pmatrix}10\\16\end{pmatrix}$ e) $\begin{pmatrix}-2\\5\end{pmatrix}$ f) $\begin{pmatrix}-3\\-13\end{pmatrix}$
6. a) $h_{\boldsymbol{a}} = -\frac{11}{7}$ b) $h_{\boldsymbol{a}} = \frac{3}{5}$ c) $h_{\boldsymbol{a}} = \sqrt{3}$
7. $x = \pm 2\sqrt{2}$
8. $h = \frac{2}{3}$, $q = -5$
9. a) 13, b) 20
   c) $\sqrt{74}$ d) $\sqrt{53}$
10. a) $a_1 = \pm 3$ b) $a_1 = \pm 8$
    c) $a_1 = \pm 8\sqrt{6}$
11. a) $\boldsymbol{e} = \begin{pmatrix}\frac{5}{13}\\\frac{12}{13}\end{pmatrix}$ eða $\boldsymbol{e} = \begin{pmatrix}-\frac{5}{13}\\-\frac{12}{13}\end{pmatrix}$
    b) $\boldsymbol{e} = \begin{pmatrix}\frac{3}{5}\\\frac{4}{5}\end{pmatrix}$ eða $\boldsymbol{e} = \begin{pmatrix}-\frac{3}{5}\\-\frac{4}{5}\end{pmatrix}$
    c) $\boldsymbol{e} = \begin{pmatrix}\frac{5}{\sqrt{74}}\\-\frac{7}{\sqrt{74}}\end{pmatrix}$ eða $\boldsymbol{e} = \begin{pmatrix}-\frac{5}{\sqrt{74}}\\\frac{7}{\sqrt{74}}\end{pmatrix}$
    d) $\boldsymbol{e} = \begin{pmatrix}\frac{2}{\sqrt{53}}\\-\frac{7}{\sqrt{53}}\end{pmatrix}$ eða $\boldsymbol{e} = \begin{pmatrix}-\frac{2}{\sqrt{53}}\\\frac{7}{\sqrt{53}}\end{pmatrix}$
12. $\begin{pmatrix}2\sqrt{2}\\-2\sqrt{2}\end{pmatrix}$ eða $\begin{pmatrix}-2\sqrt{2}\\2\sqrt{2}\end{pmatrix}$
13. $\begin{pmatrix}\frac{91}{\sqrt{313}}\\\frac{84}{\sqrt{313}}\end{pmatrix}$ eða $\begin{pmatrix}-\frac{91}{\sqrt{313}}\\-\frac{84}{\sqrt{313}}\end{pmatrix}$
14. $\begin{vmatrix}-5\\9\end{vmatrix}$
15. $\frac{1}{\sqrt{106}}\begin{vmatrix}5\\-9\end{vmatrix}$
16. $\frac{9}{5}\begin{vmatrix}4\\-3\end{vmatrix}$
17. $-5\boldsymbol{a}-4\boldsymbol{b} = \boldsymbol{c}$
18. a) $8\boldsymbol{a}-5\boldsymbol{b} = \boldsymbol{c}$ b) $-\boldsymbol{a}+\boldsymbol{b} = \boldsymbol{i}$
    c) $3\boldsymbol{a}-2\boldsymbol{b} = \boldsymbol{j}$
19. a) $\boldsymbol{a} = \begin{pmatrix}-43\\-27\end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix}-31\\-21\end{pmatrix}$
    b) $\boldsymbol{a} = \begin{pmatrix}19\\15\end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix}\frac{31}{3}\\7\end{pmatrix}$
    b) $\boldsymbol{a} = \begin{pmatrix}-50\\-36\end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix}69\\51\end{pmatrix}$
20. a) -7 b) -2 c) 9
21. a) 2 b) $\sqrt{10}$
    c) $\sqrt{15}$ d) $2\sqrt{15}$
22. a) 0 b) 16 c) -16
    d) 8 e) $\frac{527}{15}$
23. a) $t = 5$ b) Engin lausn.
    c) Engin lausn.

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24. a) $\pm\begin{pmatrix}4\\-3\end{pmatrix}$ b) $\pm\frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix}$

c) $\pm\frac{12}{\sqrt{5}}\begin{pmatrix}-1\\2\end{pmatrix}$

25. a) $\begin{pmatrix}1\\1\end{pmatrix}$ b) $\begin{pmatrix}-1\\7\end{pmatrix}$

c) $\begin{pmatrix}13\\9\end{pmatrix}$ d) $\begin{pmatrix}5\\14\end{pmatrix}$

e) $\begin{pmatrix}25\\19\end{pmatrix}$ f) $\begin{pmatrix}13-60\sqrt{2}\\91+25\sqrt{2}\end{pmatrix}$

26. a) $\overline{BP}=t\overline{BC}\Leftrightarrow$
$\overline{BO}+\overline{OP}=t\left(\overline{BO}+\overline{OC}\right)\Leftrightarrow$
$\overline{OP}=-\overline{BO}+t\overline{BO}+t\overline{OC}\Leftrightarrow$
$\overline{OP}=(1-t)\overline{OB}+t\overline{OC}$

b) P á strikinu $BC\Leftrightarrow$
$\overline{BP}=t\overline{BC}$ og $0\le t\le 1\Leftrightarrow$
$\overline{OP}=(1-t)\overline{OB}+t\overline{OC}$ og
$0\le t\le 1$ (skv. a)-lið) $\Leftrightarrow$
$\overline{OP}=s\overline{OB}+t\overline{OC}$ og $0\le t$
og $0\le s$ og $s+t=1$.

c) Ef Q er í $\triangle ABC$ skulum við kalla P skurðpunkt línunnar gegnum A og Q við hliðina BC. Skv. b)-lið fæst þá:
$\overline{OQ}=a\overline{OA}+b\overline{OP}$, $a+b=1$
$\overline{OP}=c\overline{OB}+d\overline{OC}$, $c+d=1$, þ.e.a.s.
$\overline{OQ}=a\overline{OA}+bc\overline{OB}+bd\overline{OC}$ og
$a+bc+bd=a+b(c+d)=$
$a+b\cdot 1=a+b=1$
Setjum því $s=a$, $t=bc$ og $u=bd$.

27. a) Vigrarnir $\left|\overline{AC}\right|\overline{AB}$ og $\left|\overline{AC}\right|\overline{AB}$ eru jafn langir og helmingar því summa þeirra hornið á milli þeirra, þ.e. A.

b) P er á hliðinni a (skv. dæmi 26) því að $\overline{AP}=s\overline{AB}+t\overline{AC}$ með $s+t=1$.

28. a) $t=0$ b) $t=\pm\frac{\sqrt{10}}{2}$ c) $t=0$

29. a) $\begin{pmatrix}5\\7\end{pmatrix}$ b) $\begin{pmatrix}0\\-3\end{pmatrix}$

c) $\begin{pmatrix}1\\2\end{pmatrix}$ d) $\begin{pmatrix}0\\-1\end{pmatrix}$

30. a) $\pm\frac{1}{\sqrt{74}}\begin{pmatrix}5\\7\end{pmatrix}$ b) $\pm\frac{1}{3}\begin{pmatrix}0\\-3\end{pmatrix}$

c) $\pm\frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix}$ d) $\pm\begin{pmatrix}0\\-1\end{pmatrix}$

31. a) $t=\frac{3}{8}$ b) $t=\begin{pmatrix}-1\\1.5\end{pmatrix}$

32. $B=(2,-4)$, $C=(11,-5)$

33. a) $M_a=\left(\frac{9}{2},\frac{11}{2}\right)$, $M_b=\left(3,\frac{1}{2}\right)$, $M_c=\left(\frac{1}{2},3\right)$

b) $m_a=\frac{\sqrt{346}}{2}$, $m_b=\frac{\sqrt{229}}{2}$, $m_c=\frac{13}{2}$

c) $D=(4,-7)$ d) $M=\left(3,\frac{1}{2}\right)$

34. a) $T=\left(\frac{2}{3},3\right)$

b) $M_a=\left(-2,\frac{7}{2}\right)$, $M_b=\left(1,\frac{1}{2}\right)$, $M_c=(3,5)$

c) $a=\sqrt{97}$, $b=\sqrt{109}$, $c=6\sqrt{2}$

35. $MN=\frac{\sqrt{17}}{2}$

Verkefni 2

1. a) 180° b) –45°
   c) 123.690° c) –99.4623°
2. a) –45° b) –11.3099°
   c) 125.538° d) 136.848°
3. a) $\frac{\pi}{2}$ b) $\frac{3\pi}{2}$
   c) $\frac{\pi}{6}$ d) $\frac{5\pi}{12}$
   e) 34.9066 f) –80.2852
4. a) 135° b) 114.592°
   c) 72° d) 171.887°
   e) –37.2423° f) 181.185°
5. $\cos(v)=\pm\frac{4}{5}$, $\tan(v)=\mp\frac{3}{4}$, $\cot(v)=\mp\frac{4}{3}$
6. $\cos(v)=-\frac{\sqrt{15}}{4}$, $\tan(v)=-\frac{1}{\sqrt{15}}$,
   $\cot(v)=-\sqrt{15}$
7. $\sin(v)=-\frac{12}{13}$, $\tan(v)=-\frac{12}{5}$, $\cot(v)=-\frac{5}{12}$

8. $\cos(v) = -\frac{12}{13}$, $\sin(v) = \frac{5}{13}$, $\tan(v) = -\frac{5}{12}$

9. a) –45° b) 4.57392°
c) 90° d) 0°

10. a) $\boldsymbol{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ b) $\boldsymbol{a} = \begin{pmatrix} 8 \\ 15 \end{pmatrix}$
c) $\boldsymbol{a} = \begin{pmatrix} -21 \\ 20 \end{pmatrix}$ d) $\boldsymbol{a} = \begin{pmatrix} 16 \\ 63 \end{pmatrix}$

11. $s < -2$ eða $s > 0$

12. a) $t = -6$ b) $t = -1$
c) Ekkert d) Ekkert
e) $t < -1$ eða $t > 0$ f) $t < -6$ eða $t > 0$

13. a) –132.274°
b) $\begin{vmatrix} -10 \\ -3 \end{vmatrix}$, –163.301°
c) $-\frac{10}{3} < t < 0$

14. a) –1 b) $\frac{1}{2}$ c) $-\frac{\sqrt{2}}{2}$
d) 0 e) $-\frac{1}{2}$ f) $-\frac{\sqrt{2}}{2}$
g) 1 h) $-\sqrt{3}$ i) $-\frac{1}{\sqrt{3}}$
j) –1 k) –1 l) 1

15. a) 0.283662 b) 0.996195
c) 0.5 d) –1
e) –0.544639 f) –0.707107
g) –0.871448 h) –1.14752
i) –8.14435 j) 0
k) 2.90550 l) –0.766044

16. a) 1 b) 0
c) cos(*v*) d) sin(*v*)
e) 0 f) $\frac{\sqrt{6}+\sqrt{2}}{4}$
g) 0.5 h) –1

17. Gildin í töflunni eru þessi (í sömu röð og í dæminu):

$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$
$-\frac{1}{\sqrt{3}}$	–1	$-\sqrt{3}$
$-\sqrt{3}$	–1	$-\frac{1}{\sqrt{3}}$

18. a) $-\frac{\sqrt{5}}{3}$ b) $\frac{2}{3}$
c) $\frac{2}{\sqrt{5}}$ d) $-\frac{\sqrt{5}}{2}$

19. a) $\frac{2\sqrt{91}-3\sqrt{21}}{50}$ b) $\frac{6+7\sqrt{39}}{50}$
c) $\frac{-6\sqrt{91}-91\sqrt{21}}{182+9\sqrt{39}}$ d) $\frac{-6\sqrt{91}+91\sqrt{21}}{182-9\sqrt{39}}$

20. a) $\frac{\sqrt{6}\sqrt{5+\sqrt{5}}+\sqrt{5}-1}{8}$ b) $\frac{5\sqrt{3}-3\sqrt{5}\sqrt{5-2\sqrt{5}}}{15+\sqrt{15}\sqrt{5-2\sqrt{5}}}$
c) $\frac{2\sqrt{5+\sqrt{5}}-\sqrt{2}\left(\sqrt{5}-1\right)}{8}$ d) $\frac{2\sqrt{5+\sqrt{5}}+\sqrt{2}\left(\sqrt{5}-1\right)}{2\sqrt{5+\sqrt{5}}-\sqrt{2}\left(\sqrt{5}-1\right)}$
e) $\frac{\sqrt{2}\sqrt{5+\sqrt{5}}+\sqrt{3}\left(\sqrt{5}-1\right)}{\sqrt{6}\sqrt{5+\sqrt{5}}-\sqrt{5}+1}$ f) $\frac{\sqrt{2}\sqrt{5+\sqrt{5}}}{4}$

21. a) $\sin(2v) = -\frac{120}{169}$, $\cos(2v) = -\frac{119}{169}$,
$\tan(2v) = \frac{120}{119}$, $\cot(2v) = \frac{119}{120}$
b) $\sin(\frac{v}{2}) = \pm\frac{2}{\sqrt{13}}$, $\cos(\frac{v}{2}) = \mp\frac{3}{\sqrt{13}}$,
$\tan(\frac{v}{2}) = -\frac{2}{3}$, $\cot(\frac{v}{2}) = -\frac{3}{2}$

22. $\sin(v) = \pm\frac{\sqrt{50+15\sqrt{10}}}{10}$,
$\cos(v) = \mp\frac{\sqrt{50-15\sqrt{10}}}{10}$
$\tan(v) = -3-\sqrt{10}$
$\cot(v) = -3+\sqrt{10}$

23. a) $\frac{\sqrt{9-\sqrt{6}\sqrt{5+\sqrt{5}}-\sqrt{5}}}{4}$
b) $\frac{\sqrt{8-2\sqrt{5+\sqrt{5}}-\sqrt{2}\left(\sqrt{5}-1\right)}}{4}$
c) $\frac{\sqrt{2}\sqrt{5+\sqrt{5}}+\sqrt{3}\left(\sqrt{5}-1\right)}{9+\sqrt{6}\sqrt{5+\sqrt{5}}-\sqrt{5}}$
d) $\frac{\sqrt{2}\sqrt{5+\sqrt{5}}\left(\sqrt{5}-1\right)}{8}$

24. a) 0.5cos(9*v*) + 0.5cos(*v*)
b) 0.5cos(2*v*) – 0.5cos(8*v*)
c) $-\frac{1}{4}\cos(6v) - \frac{1}{4}\cos(4v) + \frac{1}{4}\cos(2v) + \frac{1}{4}$

25. a) 2cos(3*v*)cos(*v*) b) 2cos(4*v*)sin(2*v*)
c) –2cos(5*v*+45°)sin(*v*+45°)

26. a) $-2\sin^2(v)$
b) 4cos(*v* + 15°sin(*v* – 15°)
c) $2\cos(\frac{v}{2}+30°)\cos(\frac{v}{2}-30°)$

4 Svör við verkefnum

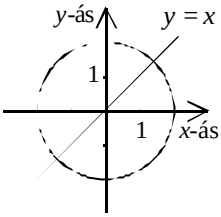
Verkefni 3

1. a) $A = 71.8109°$, $B = 46.6847°$, $C = 61.5044°$, $a = \sqrt{104}$, $b = \sqrt{61}$, $c = \sqrt{89}$
   b) $a = \sqrt{185}$, $b = \sqrt{130}$, $c = \sqrt{73}$, $A = 84.6991°$, $B = 56.5834°$, $C = 38.7175°$
   c) $a = \sqrt{116}$, $b = \sqrt{20}$, $c = 8$, $A = 116.5651°$, $B = 21.8014°$, $C = 41.6335°$
2. a) 35 b) 48.5 c) 16
3. a) $A = 50.7035°$, $B = 33.5573°$, $C = 95.7392°$
   b) $B = 49.4941°$, $C = 35.5059°$, $c = 5.39281$
   c) $C = 92.23°$, $a = 9.80811$, $b = 12.8923$
   d) $A = 46.3452°$, $C = 77.2048°$, $b = 26.4934$
   e) $B = 46.3140°$, $C = 93.6860°$, $c = 11.0400$
   eða
   $B = 133.6860°$, $C = 6.31401°$, $c = 1.36876$
4. a) $a = 4\sqrt{26}$, $b = 13$ og $c = 17$
   b) $A = 84.5474°$, $B = 39.3824°$, $C = 56.0702°$
   c) 110 d) $T = \left(\frac{14}{3}, -\frac{5}{3}\right)$
   e) $D = (0,-0.2)$
   f) $P = \left(0, 2\sqrt{10}\right)$, $Q = \left(0, -2\sqrt{10}\right)$
5. a) i) $(-13,-18)$, ii) $(-20,5)$,
   iii) $\left(-5 - \frac{7\sqrt{2}}{2}, -3 + \frac{23\sqrt{2}}{2}\right)$
   iv) $\left(-\frac{25}{2} + 4\sqrt{3}, 1 + \frac{15\sqrt{3}}{2}\right)$
   v) $(-8.51346, 13.6330)$
   vi) $(7.64784, 8.35923)$
   b) $\begin{bmatrix} 8 \\ 15 \end{bmatrix} = \begin{bmatrix} \frac{23}{2} \\ \frac{23}{2} \end{bmatrix} + \begin{bmatrix} -\frac{7}{2} \\ \frac{7}{2} \end{bmatrix}$
   c) $\boldsymbol{a} = \begin{pmatrix} \frac{23}{2} \\ \frac{7}{2} \end{pmatrix}$
   d) $B = (6.5,0.5)$, $D = (-8.5,8.5)$
6. a) -3 b) 2
   c) 82 d) 0
7. a) 35 b) 48.5 c) 16
8. a) $x = 1$ og $y = -2$
   b) $x = -\frac{15}{11}$ og $y = \frac{9}{22}$
   c) $x = -48$ og $y = -70$
   d) Ef $a^2 \neq 4$ er $x = \frac{5}{2(a-2)}$ og $y = \frac{5}{a-2}$.
   Ef $a = 2$ er engin lausn.
   Ef $a = -2$ er $x = t$ og $y = -2t - \frac{5}{2}$ og $t \in \mathbb{R}$.
9. a) $\begin{bmatrix} 2 \\ 4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0\begin{bmatrix} 1 \\ -2 \end{bmatrix}$
   b) $\begin{bmatrix} 2 \\ 4 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 4 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 3 \end{bmatrix}$
   c) $\begin{bmatrix} 2 \\ 4 \end{bmatrix} = -6\begin{bmatrix} 1 \\ 6 \end{bmatrix} + 8\begin{bmatrix} 1 \\ 5 \end{bmatrix}$
10. a) $a = 5\sqrt{13}$, $b = 15$, $c = 10$
    b) $86.8202°$, $233.1301°$ og $-36.8699°$
    c) $s = t = -1$, d) 75, e) $D = (2,16)$
    f) i) $(-7,4)$, ii) $(7,6)$
    iii) $(10.89950, -0.585787)$
    iv) $(8.47449, 4.64319)$
11. 22
12. $\overline{AB} = \frac{3}{2}\overline{DC}$
13. Sönnun, svari sleppt.
14. $-26.5651°$ eða $153.4350°$
15. Sönnun, svari sleppt.
16. $$\begin{aligned}(TCA) &= \frac{1}{2}\left|\det\left(\overline{TC}, \overline{TA}\right)\right| \\ &= \frac{1}{2}\left|\det\left(\overline{TC}, -\overline{TB} - \overline{TC}\right)\right| \\ &= \frac{1}{2}\left|\det\left(\overline{TC}, -\overline{TB}\right)\right| \\ &= \frac{1}{2}\left|\det\left(\overline{TB}, \overline{TC}\right)\right| = (TBC)\end{aligned}$$
    Á sama hátt fæst að $(TBC) = (TBA)$.

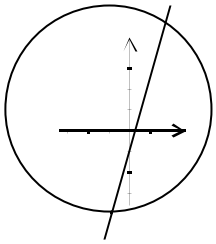
(Hér var notuð niðurstaðan í dæmi 1 í æfingu 1.2B.)

Verkefni 4

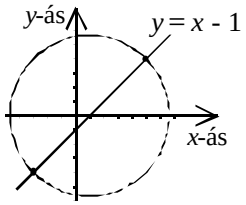
1. a) $(x-2)^2+(y-4)^2=5^2$
   b) $x^2+(y+1)^2=7^2$
   c) $x^2+y^2=6$
   d) $(x+5)^2+(y-3.5)^2=1^2$
   e) $(x+3)^2+(y+7)^2=17^2$
   f) $\left(x-\frac{2}{3}\right)^2+\left(y+\frac{5}{11}\right)^2=13$
2. a) $x^2+y^2=5^2$
   b) $(x-1)^2+(y-1)^2=5^2$
   c) $(x-4)^2+(y-5)^2=5^2$
   d) $(x-14)^2+(y-6)^2=13^2$
   e) $(x+6)^2+(y+3)^2=5^2$
   f) $(x-3)^2+(y+5)^2=9^2$
   g) $(x-2)^2+(y+2)^2=40$
3. a) $x^2+(y-5)^2=5^2$
   b) $(x-3)^2+(y-4)^2=5^2$
   c) $x^2+(y-5)^2=5^2$
   d) $x^2+(y-5.5)^2=6.5^2$
   e) $(x+2)^2+(y-1)^2=5$
4. a) $C=(5,-2),\ r=3$
   b) $C=(3,-1),\ r=2$
   c) $C=(-2,-3),\ r=\sqrt{5}$
   d) $C=(0,3),\ r=1$
   e) $C=\left(-\frac{1}{2},0\right),\ r=3$
5. a) $S_1=\left(-\sqrt{2},-\sqrt{2}\right),\ S_2=\left(\sqrt{2},\sqrt{2}\right)$



b) $S_1=(-1,-4),\ S_2=(2,5)$

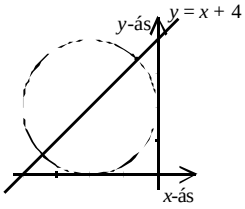


c) $S_1=(-3,-4),\ S_2=(5,4)$

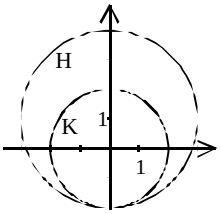


d) $S_1=\left(-2+2\sqrt{2},2+2\sqrt{2}\right)$

$S_2=\left(-2-2\sqrt{2},2-2\sqrt{2}\right)$



6.a) $S_1=(0,-2)$



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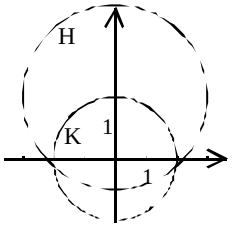
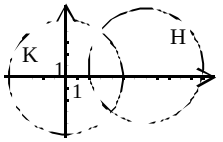
b) $S_1=\left(-\frac{3\sqrt{7}}{4},-\frac{1}{4}\right)$, $S_2=\left(\frac{3\sqrt{7}}{4},-\frac{1}{4}\right)$



c) $S_1=(3,4)$,

$S_2=(4,-3)$

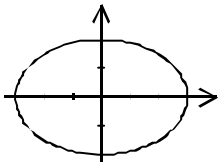


7. $(x-3)^2+(y-4)^2=25$

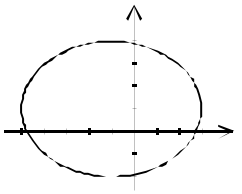
eða

$(x+3)^2+(y+4)^2=25$

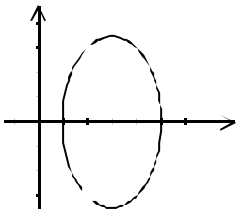
8.a) $C=(0,0)$, $2a=6$, $2b=4$



b) $C=(-1,1)$, $2a=8$, $2b=6$



c) $C=(3,0)$, $2a=4\sqrt{3}$, $2b=4$



d) $C=(2.5,-2)$, $2a=8$, $2b=4\sqrt{2}$

9. a) $F_1=\left(-\sqrt{5},0\right)$, $F_2=\left(\sqrt{5},0\right)$, $e=\frac{\sqrt{5}}{3}$

b) $F_1=\left(-1-\sqrt{7},1\right)$, $F_2=\left(-1+\sqrt{7},1\right)$,

$e=\frac{\sqrt{7}}{4}$

c) $F_1=\left(3,-2\sqrt{2}\right)$, $F_2=\left(3,2\sqrt{2}\right)$,

$e=\frac{\sqrt{2}}{\sqrt{3}}$

d) $F_1=\left(2.5,-2-2\sqrt{2}\right)$,

$F_2=\left(2.5,-2+2\sqrt{2}\right)$, $e=\frac{\sqrt{2}}{2}$

10.a) $\frac{(x-4)^2}{5^2}+\frac{(y-1)^2}{4^2}=1$

b) $\frac{(x-2)^2}{3^2}+\frac{(y+1)^2}{6^2}=1$

11.a) $\frac{x^2}{5^2}+\frac{y^2}{4^2}=1$

$C=(0,0)$, $2a=10$, $2b=8$,

$F_1=(-3,0)$, $F_2=(3,0)$, $e=\frac{3}{5}$

b) $\frac{x^2}{4^2}+\frac{y^2}{(\sqrt{7})^2}=1$

$C=(0,0)$, $2a=8$, $2b=2\sqrt{7}$,

$F_1=(-3,0)$, $F_2=(3,0)$, $e=\frac{3}{4}$

c) $\frac{x^2}{2}+\frac{y^2}{3}=1$

$C=(0,0)$, $2a=2\sqrt{3}$, $2b=2\sqrt{2}$,

$F_1=(0,-1)$, $F_2=(0,1)$, $e=\frac{\sqrt{3}}{3}$

d) $\frac{x^2}{2^2}+\frac{(y+1)^2}{1^2}=1$

$C=(0,-1)$, $2a=4$, $2b=2$,

$F_1=\left(-\sqrt{3},-1\right)$, $F_2=\left(\sqrt{3},-1\right)$, $e=\frac{\sqrt{3}}{2}$

e) $\frac{(x-1)^2}{2^2}+\frac{(y+2)^2}{3^2}=1$

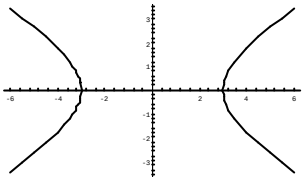
$C=(1,-2)$, $2a=6$, $2b=4$,

$F_1=\left(1,-2-\sqrt{5}\right)$, $F_2=\left(1,-2+\sqrt{5}\right)$,

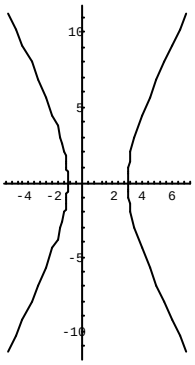
$e = \frac{\sqrt{5}}{3}$

12. Mest 35.60 AU, minnst 0.58 AU.
13. $e = 0.0523$
14. a) $F = 18.8496$ b) $F = 37.6991$
    c) $F = 21.7656$ d) $F = 35.5431$

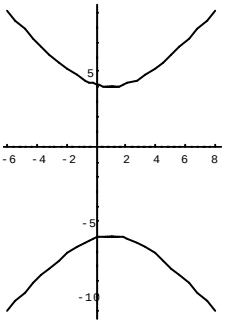
15.a)



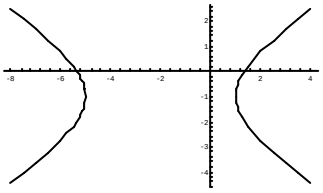
b)



c)



d)



16.a) $C = (3,-1)$, $T_1 = (1,-1)$, $T_2 = (5,-1)$
b) $C = (0,0)$, $T_1 = (-0.5,0)$, $T_2 = (0.5,0)$
c) $C = (0,0)$, $T_1 = (-9,0)$, $T_2 = (9,0)$
d) $C = (7,2)$, $T_1 = (2,2)$, $T_2 = (12,2)$
e) $C = (-1,6)$, $T_1 = (-1,4)$, $T_2 = (-1,8)$
f) $C = (3,-2)$, $T_1 = (3,-5)$, $T_2 = (3,1)$

17.a) Sporbaugur b) Fleygbogi
c) Breiðbogi d) Breiðbogi
e) Sporbaugur f) Breiðbogi

18. $r \geq \frac{3\sqrt{5}}{5}$
19. $2\sqrt{3}$
20. Stærst $2a$, minnst $2b$.

Verkefni 5

1. a) $x = -1 + 6\cos(v)$, $y = 3 + 6\sin(v)$
   b) $x = 1 + \cos(v)$, $y = -0.5 + \sin(v)$
   c) $x = -3 + 5\cos(v)$, $y = -2 + 5\sin(v)$
2. $S_1 = \left(1 + \frac{4\sqrt{5}}{5}, 2 - \frac{2\sqrt{5}}{5}\right)$
   $S_2 = \left(1 - \frac{4\sqrt{5}}{5}, 2 + \frac{2\sqrt{5}}{5}\right)$
3. $S_1 = \left(\frac{-13-\sqrt{39}}{10}, \frac{1-3\sqrt{39}}{10}\right)$
   $S_2 = \left(\frac{-13+\sqrt{39}}{10}, \frac{1+3\sqrt{39}}{10}\right)$
4. a: $3x + 2y - 24 = 0$
   b: $7x - 6y + 8 = 0$
   c: $x - 2y = 0$
5. $S = \left(\frac{26}{7}, \frac{53}{7}\right)$
6. a: $x = 6 + 7t$, $y = 4 - t$
   b: $x = 2 - 3t$, $y = -5 + 10t$
   c: $x = 2 + 4t$, $y = -5 + 9t$
7. a: $x = 4 + 2t$, $y = 6 - 3t$
   b: $x = -2 + 6t$, $y = -1 + 7t$
   c: $x = -2 + 2t$, $y = -1 + t$
8. $x + 2y - 17 = 0$ ef $x \geq 5$
   $x - 2y + 7 = 0$ ef $x < 5$
9. $S = \left(\frac{18}{7}, \frac{29}{7}\right)$
10. $S = \left(\frac{15}{13}, -\frac{55}{13}\right)$
11. a) $p = -3$ og $y_0 = -\frac{2}{3}$

b) $p = -3$ og $y_0 \neq -\frac{2}{3}$

12. $S_1 = \left(\frac{-9-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$, $S_2 = \left(\frac{-9+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}\right)$

13. $80x + 23y - 115 = 0$

14. $(x + 2)^2 + y^2 = 20$

15. a) $x^2 + y^2 = 65^2$

b) $(x + 3)^2 + (y - 4)^2 = 65^2$

16. $\pm\frac{63}{5}$, $\frac{63}{25}$ 

17. $C_1 = \left(\frac{506}{97}, \frac{217}{97}\right)$, $h_c = \frac{67}{\sqrt{97}}$

18. a) $\pm\frac{14}{\sqrt{29}}$ b) $-\frac{14}{29}$ 

c) $A_1 = \left(\frac{117}{29}, \frac{12}{29}\right)$ d) $-\frac{14}{\sqrt{29}}$

19. a) $\frac{64}{\sqrt{34}}$ b) 7

c) 1 d) 0

20. a) $-5x + 12y - 58 = 0$

$-5x + 12y + 72 = 0$

b) $x + 2y + 5 - 5\sqrt{5} = 0$

$x + 2y + 5 + 5\sqrt{5} = 0$

21. $x + 7y - 70 = 0$, $7x - y - 12 = 0$

22. $r \geq \frac{3\sqrt{5}}{5}$

23. $3\sqrt{5} - 5$

24. 7

25. $\frac{125}{12}$

26. $2\sqrt{3}$

27. v_A: $0.734263x + 19.11904y - 34.4142 = 0$

v_B: $29.0122x + 10.6274y - 84.1665 = 0$

v_C: $182.8841x - 93.5107y - 72.8241 = 0$

Verkefni 6

1. a) $v = 90° + h\cdot360°$ eða
$v = -150° + h\cdot360°$
b) $v = 19.6087° + h\cdot45°$
2. a) $v = \pm120° + h\cdot360°$ eða
$v = \pm109.471° + h\cdot360°$
b) $v = \pm54.7356° + h\cdot180°$
c) $v = h\cdot180°$ eða
$v = \pm35.2544° + h\cdot360°$ eða
$v = \pm144.7356° + h\cdot360°$
3. a) $v = -45° + h\cdot180°$ eða
$v = 18.4349° + h\cdot180°$
b) $v = 31.7175° + h\cdot180°$ eða
$v = -58.2825° + h\cdot180°$
4. a) $v = 90° + h\cdot360°$ eða
$v = \left(\frac{90}{7}\right)° + h\cdot\frac{360°}{7}$
b) $v = 14° + h\cdot72°$ eða
$v = -\left(\frac{70}{3}\right)° + h\cdot120°$
5. a) $v = 55.1988° + h\cdot360°$ eða
$v = 170.041° + h\cdot360°$
b) $v = -45° + h\cdot180°$ eða
$v = 90° + h\cdot180°$
6. a) $v = h\cdot360°$ eða
$v = \pm120° + h\cdot360°$
b) $v = 90° + h\cdot180°$ eða
$v = -131.410° + h\cdot360°$ eða
$v = -48.5904° + h\cdot360°$
c) $v = 90° + h\cdot180°$ eða
$v = \pm60° + h\cdot360°$ eða
$v = \pm120° + h\cdot360°$
d) $v = \frac{2p}{15} + h\cdot 2p$ eða $v = -\frac{p}{3} + h\cdot 2p$

Verkefni 7

1. a) 0 1 0 0 b) 1 0 0 0 c) 0 0 1 0
d) 1 0 1 1 e) 0 1 1 0 f) 1 1 1 1
2. a) $x \leq 1$ b) $x \leq 2 \vee x \geq 4$
c) $2 < x < 4$ d) $1 < x$
e) $x \in \varnothing$ f) $x \leq 1$
g) $x \leq 1 \vee 2 < x < 4$ h) $x \leq 2 \vee x \geq 4$
3. a) $x^4 - 18x^3 + 99x^2 - 162x = 0$
b) $x \in \mathbb{Z}_+$ og 3^x c) $(x+4)(x+7) = 0$
d) $x^2 + 1 = 0$ e) $n \in \mathbb{Z}$ og $x = n$
4. a) $\mathbb{R}$ b) $\varnothing$ c) $\mathbb{R} \setminus \{0\}$
d) $\{0,1\}$ e) $\{0\}$
5. a) Fyrir allar neikvæðar heilar tölur gildir að annað veldi þeirra er stærra en talan sjálf. Sanngildið 1.

b) Það er til jákvæð heil tala sem er þannig að þriðja veldi hennar er minna en 1. Sanngildið 0.

6. a) $\exists x \in \mathbb{Z}_+ : x = x^5$, Sanngildið 1.
   b) $\forall x \in \mathbb{R} \setminus \mathbb{Q} \exists y \in \mathbb{Z} : y > x$, Sanngildið 1.
   c) $\exists x \in \mathbb{Z} \forall y \in \mathbb{R} \setminus \mathbb{Q} : x > y$, Sanngildið 0.

Verkefni 8

1.
2.
3.
4.
5.
5. a) 25401600 b) 7315660800
6. a) 1140, b) $4.0548 \cdot 10^{17}$
   c) 27713400
7. 22
8. 23629008
9. a) 270725 b) 715
   c) 495 d) 1
   e) 2860 f) 267865
   g) 28561 h) 36504
   i) 81120
10. a) 360360 b) 353192
11.
12.
13.
14.