

Svör

Svör við verkefnum

Verkefni 1

1. a), b), f) og g).

2. a) \mathbb{R} b) \mathbb{R} c) $\mathbb{R} \setminus \{-1\}$

d) \mathbb{R} e) $\mathbb{R} \setminus \{0,4\}$ f) $\mathbb{R} \setminus \{1\}$

g) $\left[\frac{1}{3}, \infty\right[$ h) $\left]\frac{1}{3}, \infty\right[$

3. a) \mathbb{R} b) $\mathbb{R} \setminus \{0\}$ c) \mathbb{R}

d) $\mathbb{R} \setminus \{-3\}$ e) $[-3, \infty[$ f) $]-3, \infty[$

g) $]-\infty, 3]$ h) $\left[\frac{5}{2}, \infty\right[$

4. a) $\{-1, 1, 3, 5, 7\}$

b) $\{-11, -8, -5, -2\}$

c) $\{-3, 0, 1\}$

d) $]-\infty, 1]$

e) $\left\{3, 2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}\right\}$

f) $]1, \infty[$

5. a) $[5, \infty[$ b) \mathbb{R} c) $]-\infty, 1]$

d) $\mathbb{R} \setminus \{3\}$

6. a) $D_f = V_f = \mathbb{R} \setminus \{1\}$

b) $D_f = \mathbb{R} \setminus \{2\}, V_f = \mathbb{R} \setminus \{1\}$

c) $D_f = \mathbb{R} \setminus \{1\}, V_f = \mathbb{R} \setminus \{2\}$

d) $D_f = \mathbb{R} \setminus \{-1\}, V_f = \mathbb{R} \setminus \{1\}$

e) $D_f = \mathbb{R} \setminus \{-4\}, V_f = \mathbb{R} \setminus \{5\}$

f) $D_f = \mathbb{R} \setminus \{-2\}, V_f = \mathbb{R} \setminus \{-3\}$

7. a) $\{-2, 1\}$ b) $1 \pm \sqrt{3}$

c) $2 \pm \sqrt{3}$ d) $\{0, 2\}$

8. a) 3 b) $\frac{3}{4}$ c) $[-13, 2]$

9. a) $\pm\sqrt{17}$ b) $\{-3, 4\}$

10. a) $\{-5, 6\}$ b) $\{-1, 3\}$ c) $\left[-\frac{13}{4}, 9\right]$

11. a) $[1, \infty[$ b) $[0, 2]$ c) $\mathbb{R}_+ \cup \{0\}$

12. a) $(f + g)(x) = x^3 + 2x^2$

$(f - g)(x) = x^3 - 2x^2 - 2x$

$(f \cdot g)(x) = 2x^5 + x^4 - 2x^3 - x^2$

$(f : g)(x) = \frac{x^2 - 1}{2x + 1}$

b) $(f + g)(x) = \frac{x^2 + x - 3}{x^2 - 3x}$

$(f - g)(x) = \frac{-x^2 + x - 3}{x^2 - 3x}$

$(f \cdot g)(x) = \frac{1}{x - 3}$

$(f : g)(x) = \frac{x - 3}{x^2}$

c) $(f + g)(x) = 3\sqrt{x} - 1$

$(f - g)(x) = \sqrt{x} + 1$

$(f \cdot g)(x) = 2x - 2\sqrt{x}$

$(f : g)(x) = \frac{2\sqrt{x}}{\sqrt{x} - 1}$ og $x \neq 1$

d) $(f + g)(x) = \frac{5x^2 + 5}{(x - 1)(3x + 2)}$

$(f - g)(x) = \frac{x^2 + 10x - 1}{(x - 1)(3x + 2)}$

$(f \cdot g)(x) = \frac{(x + 1)(2x - 3)}{(x - 1)(3x + 2)}$

$(f : g)(x) = \frac{(x + 1)(3x + 2)}{(x - 1)(2x - 3)}$

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13. $y = 100 + \frac{5000}{x}$

14. Sönnun, svari sleppt.

15. Sönnun, svari sleppt.

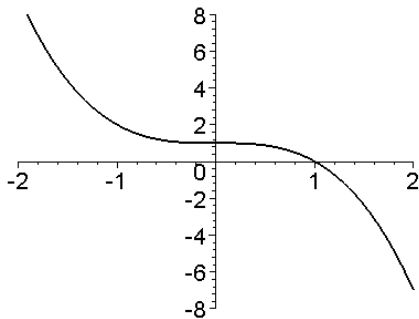
16. $f(x) = \frac{1-2x}{2(1-x)}, 0 \leq x < 0.5$

$g(x) = \frac{x(1-2x)}{4(1-x)}, 0 \leq x < 0.5$

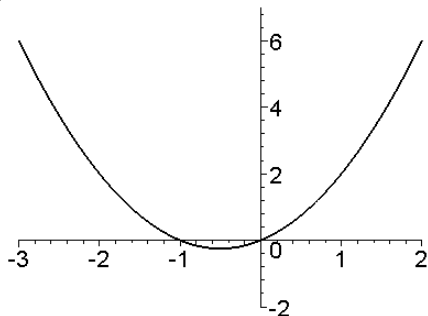
$h(x) = \frac{2x^2 - 2x + 1}{4(1-x)}, 0 \leq x < 0.5$

$j(x) = \frac{x(1-2x)}{2(1-x)}, 0 \leq x < 0.5$

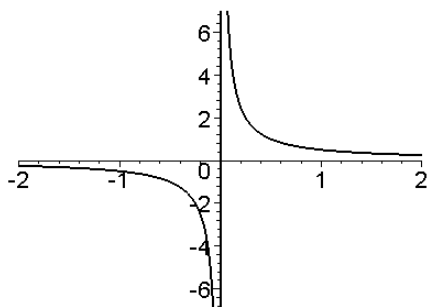
17.a)



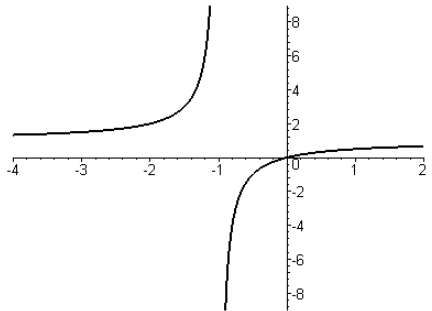
b)



c)



d)



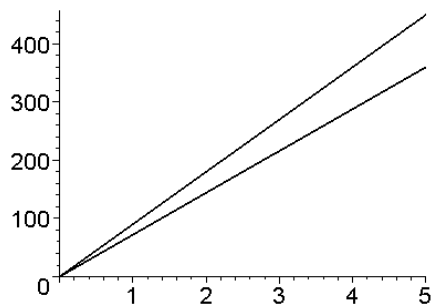
18.a) $D_f = [-2, 2]$

b) $f(-1) = 4, f(0) = 1, f(2) = 8$

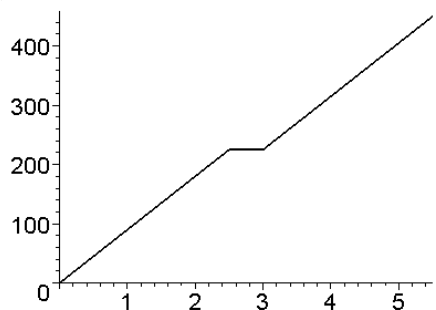
c) $x \in \left\{ -\frac{6}{5}, -\frac{4}{5}, 1 \right\}$

d) $V_f = [-1, 8]$ e) $f(M) = [1, 4]$

19.a) $x = 72t$



b)



20.a) $g(x) = x^2 + 2x + 2$

b) $g(x) = \sqrt{x-1} + 7$

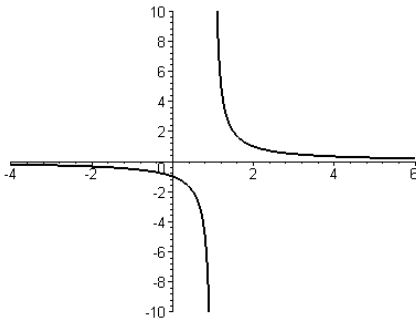
c) $g(x) = \sqrt{x+4} + 3$

d) $g(x) = \frac{x-1}{x-3} + 3$

e) $g(x) = \frac{x^2 - 4x + 5}{x^2 - 4x + 3} + 3$

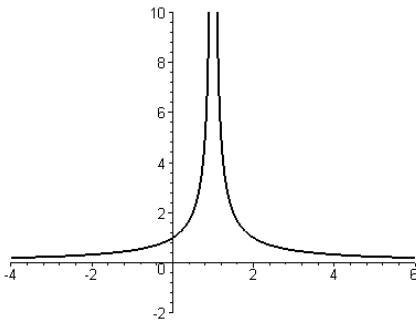
21. $g(x) = \frac{1}{x-1}$ fæst með því að hliðra

$f(x) = \frac{1}{x}$ um $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.



$h(x) = \left| \frac{1}{x-1} \right|$ fæst með því að spegla

$g(x) = \frac{1}{x-1}$ um x -ás fyrir $x < 1$, annars óbreytt.



$\left[0, \frac{1}{2} \right] \cup \left[\frac{3}{2}, 2 \right]$

22. $0 < \frac{1}{x^2 + 1} \leq 1$, $k = 0$, $K = 1$,

23.a) Hágildi $(-3, 8)$ og $(3, 8)$
Lágildi $(0, -1)$

b) Hágildi $(-3, 64)$ og $(3, 64)$

Lágildi $(-1, 0)$ og $(1, 0)$

c) Hágildi $(-3, 36)$

Lágildi $(3, -18)$

24.a) Ummálið er $U = 2h + x(\sqrt{2} + 1) = 12$ en

flatarmálið er $F = xh + \frac{x^2}{4}$. Með því að

leysa fyrri jöfnuna m.t.t. h og setja inn í formúluna fyrir F fæst niðurstaðan.

b) $x \approx 3.13$ m $F(3.13) \approx 9.40334$ m²

25.a) $x_2 - x_1 > 0$

$f(x_2) - f(x_1) = \frac{3}{2}(x_2 - x_1) > 0$

b) $x_2 > x_1 > 1$

$f(x_2) - f(x_1) = 2(x_2 - x_1)(x_2 + x_1 - 2) > 0$

c) $x_2 - x_1 > 0$

$f(x_2) - f(x_1) = \frac{x_2 - x_1}{\sqrt{x_2 - 1} + \sqrt{x_1 - 1}} > 0$

d) Ef $x \geq 1$ er $f(x) = x + 1$

Ef $x < 1$ er $f(x) = 3x - 1$

Í báðum tilvikum er f einhalla vaxandi.

26.a) $\left] -\infty, -\frac{5}{2} \right]$ b) $[0, \infty[$

27.a) Summa tveggja vaxandi falla.

b) $f(-x) = \sqrt{1+x^2} - x$

$= \left(\sqrt{1+x^2} - x \right) \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2} + x}$

$= \frac{1}{\sqrt{1+x^2} + x} = \frac{1}{f(x)}$

c) Rökstuðningurinn er í svari við b).

d) Rökstuðningurinn liggur í a) og c).

e) Sönnun, svari sleppt.

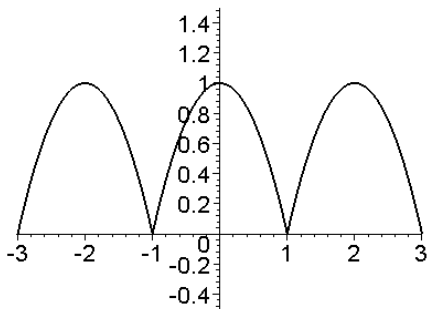
28.a) hvorugt b) jafnt

c) ójafnt. d) ójafnt

e) ójafnt f) hvorugt

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29.



$$b) D_f = \mathbb{R} \setminus \left\{ \frac{1}{2} \right\} \quad V_f = \mathbb{R} \setminus \left\{ \frac{5}{2} \right\}$$

$$f^{-1}(x) = \frac{x-3}{2x-5}$$

$$36.a) \frac{8-x}{x+1} \quad b) \frac{x}{3x-1}$$

$$c) \frac{x}{3-x} \quad d) \frac{3}{x+1}$$

$$e) \frac{x}{3-x} \quad f) \frac{1}{3-x}$$

$$30.a) 27 \quad b) \sqrt{13} - 2$$

$$c) \sqrt{x+2} \quad d) \sqrt[4]{x} - 2$$

$$31.a) \frac{7x+20}{x+35} \quad b) \frac{35-8x}{11x-20}$$

$$32.a) 5x^2 - 58$$

$$b) 25x^2 - 30x - 2$$

$$c) 45x^2 - 150x + 67$$

$$d) 950$$

$$e) 67$$

$$f) -27x^2 + 90x - 37$$

$$33.a) \frac{1}{x} \quad b) \frac{1}{x}$$

$$c) 1 - \frac{1}{x} \quad d) \frac{1}{1-x}$$

$$e) \frac{x}{x-1} \quad f) \frac{x}{x-1}$$

$$g) 1 - \frac{1}{x} \quad h) \frac{1}{1-x}$$

$$34.a) f^{-1}(x) = \frac{1-x}{4}$$

$$b) f^{-1}(x) = \frac{x-2}{3}$$

$$c) \frac{-3 + \sqrt{4x+1}}{2}, x \geq 2$$

$$d) \frac{-3 + \sqrt{4x+1}}{2}, x > 0$$

$$35.a) D_f = \mathbb{R} \setminus \{-1\} \quad V_f = \mathbb{R} \setminus \{2\}$$

$$f^{-1}(x) = \frac{x}{2-x}$$

Verkefni 2

$$1. a) [4,8] \quad b) \emptyset$$

$$c)]-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty[$$

$$2. a) \text{tvöfaldast} \quad b) \text{áttfaldast}$$

$$3. a) x \in \left\{ -\frac{1}{2}, \frac{7}{2} \right\} \quad b) \emptyset$$

$$c) x \in \left\{ -\frac{2}{3}, \frac{4}{3} \right\}$$

$$4. a) x \in \{-1, 1\} \quad b) x \in \{-1, 1\}$$

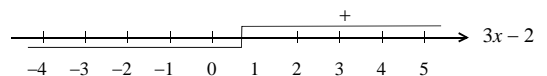
$$c) x \in \{1, \sqrt{2}\} \quad d) x = \sqrt[3]{2}$$

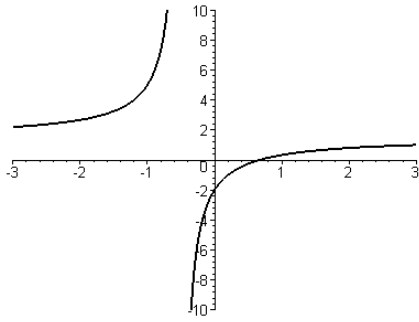
$$5. a) \left] \frac{1-\sqrt{29}}{2}, \frac{1-\sqrt{13}}{2} \right[\cup \left] \frac{1+\sqrt{13}}{2}, \frac{1+\sqrt{29}}{2} \right[$$

$$b) \left] \frac{1-\sqrt{5}}{2}, 0 \right[\cup \left] 1, \frac{1+\sqrt{5}}{2} \right[$$

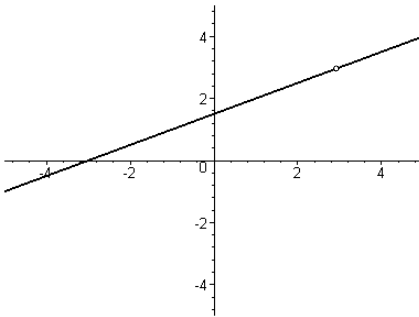
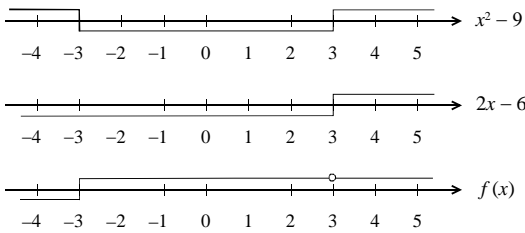
$$c) \left] \frac{6-\sqrt{40}}{2}, 1 \right[$$

6. a)

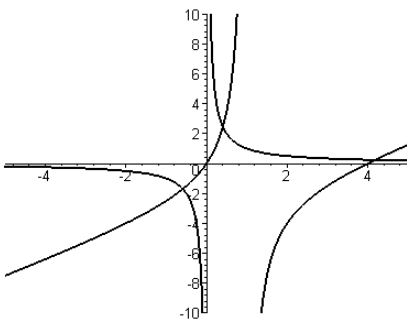




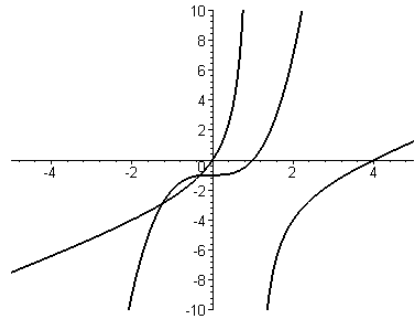
b)



7. a)

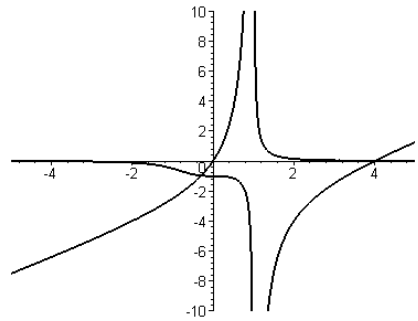


b)



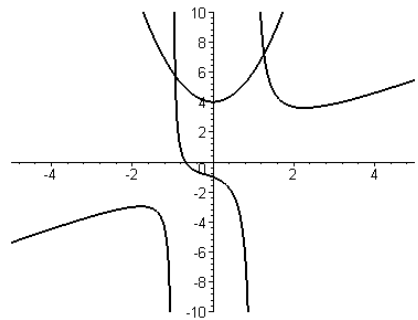
$$x \in]-1.23803, -0.31403[\cup]1, \infty[$$

c)



$$x \in]-\infty, -0.29387[\cup]1, 4.01181[$$

d)



$$x \in]-\infty, -1[\cup]-0.93288, 1[\cup]1.26289, \infty[$$

$$x \in]-\infty, -0.58836[\cup]0, 0.40642[\cup]1, 4.18194[$$

8. 6885.64 kr

9. 8.664 ár.

10.a) $x = \sqrt{2}$

b) $x = \sqrt{2}$

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c) $x = 10$

d) $x = \frac{1}{\log(2)}$

e) $x = \frac{\log(20)}{\log(2)}$

f) $x = \frac{\ln(2 \pm \sqrt{3})}{\ln(2)} = \begin{cases} 1.89997 \\ -1.89997 \end{cases}$

g) $x = \begin{cases} 0 \\ \frac{\ln(3)}{\ln(4)} \end{cases}$

11.a) $y = \frac{e^{2x-1} - 1}{2}$ b) $y = \frac{e^{x^2-1} - 1}{2}$

12.a) $y = \frac{1}{2} \cdot \ln(x+1) + \ln(\sqrt{2}) - 1$

b) $y = \frac{1}{2} \cdot (\ln(x+1) + \ln(x-1) + 1)$

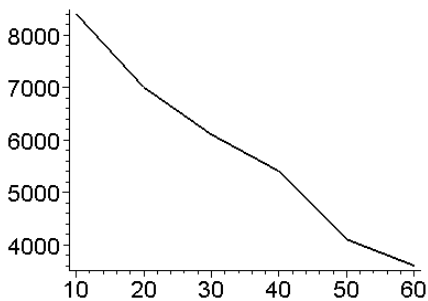
13.871 ár.

14.a) 40.5%

b) 2075 dagar.

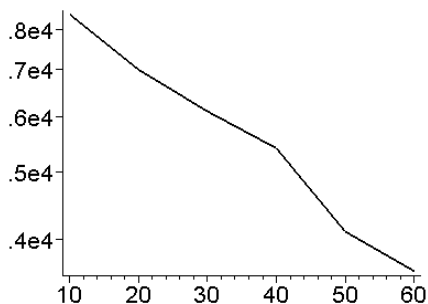
15.34.5 sek.

16.a)



$T = 40$ dagar

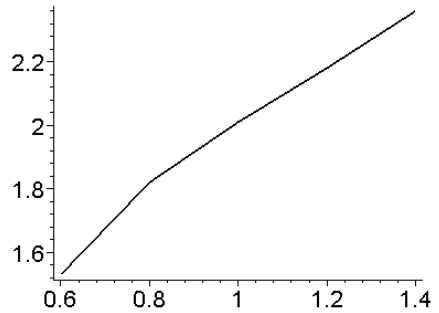
b)



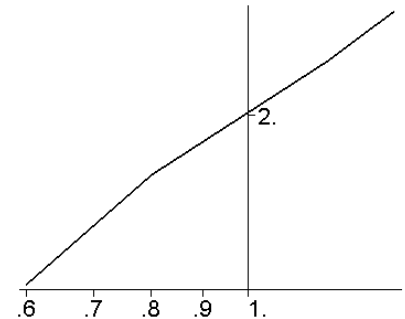
17.2.005 sek.

18.0.248 m.

19.a)



b)



20.a) $g = 1.51582$

b) $g = 1.26036$

c) $g = 1.24449$

d) $g = 4551.03$

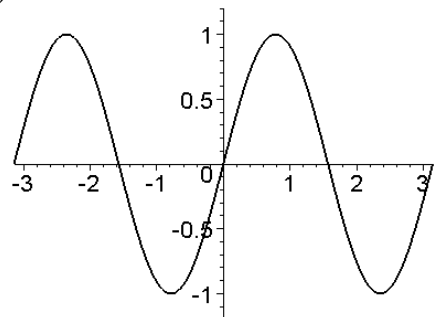
e) $g = 17614.95$

f) $g = 0.803545$

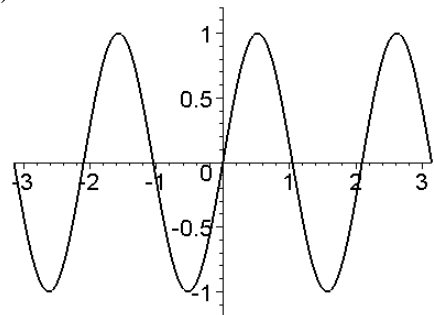
21.a) $x = 8.6987$

b) $x = 1.$

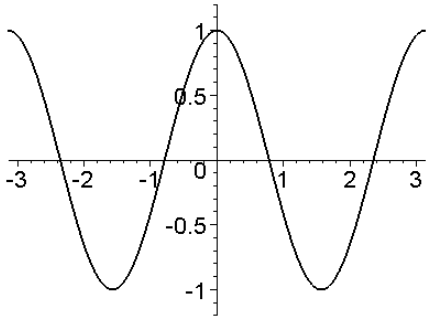
22.a)



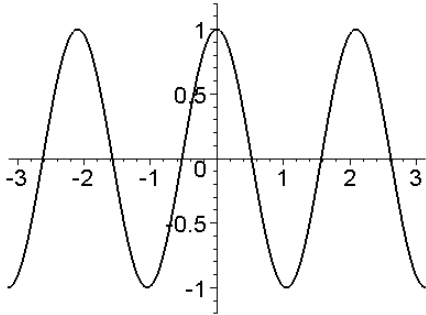
b)



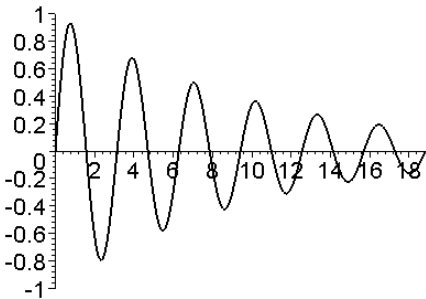
c)



d)



23. $y = e^{-0.1t} \sin(2t)$



Verkefni 3

1. a) 7 b) 0 c) -3.
d) 2.
2. a) -4 b) 3 c) 2
d) 0.
3. a) Já, markgildið er -2
4. a) $\frac{5}{6}$ b) $\frac{4}{5}$ c) 32
d) 2.
5. a) e^{-2} b) \sqrt{e} c) 0
d) 1 e) 1.
6. a) 0 b) 1 c) 0.

7. a) $f(2) = \frac{3}{4}$.

8. $a = 4$.

9. $f(-2) < 0, f(0) > 0, f(1) < 0, f(3) > 0$ og $x \in \{-1.48, 0.31, 2.17\}$

10. a) -3 b) 9 c) 0 d) 0
e) -1 f) $\frac{1}{2}$

11. a) ∞ b) -2 c) $\frac{7}{8}$ d) $\frac{7}{8}$

12. a) 0 b) $\frac{3}{8}$ c) $\frac{3}{8}$ d) $\frac{4}{15}$
e) 0 f) 0

13. a) $-\frac{1}{6}$ b) $-\frac{5}{2}$ c) -1

14. a) $x = 3$ og $y = 0$ b) $x = -5$ og $y = 1$
c) $x = \pm 2$ og $y = 0$ b) $x = -1$ og $y = 3$

15. a) $y = x + 2$ og $x = 1$ b) $y = x$ og $x = -1$
c) $x = \pm 3$ og $y = 2x$.

16. a) $y = 2$ er aðfella í hægri enda.
b) $x = \sqrt{2}$ er aðfella í neðri enda.

- c) Línurnar $x = \frac{\pi}{4} + h \frac{\pi}{2} \wedge h \in \mathbb{Z}$, eru aðfellar í báða enda.

17. a) $y = x$ er aðfella upp til hægri, og $y = -x$ er aðfella upp til vinstri.

- b) $y = 2x$ er aðfella upp til hægri, en $y = -2x$ er aðfella upp til vinstri.

- c) $y = 0$ er aðfella til hægri.

- d) $y = x - \frac{1}{2}$ er aðfella upp til hægri, en $y = -x + \frac{1}{2}$ er aðfella upp til vinstri.

Verkefni 4

1. a) $y = 3x - 9$.

2. a) $y = -4x + 4$. b) $y = 5x - 3$

c) $y = 3x - 3$.

d) $y = \frac{\sqrt{6}}{4}x + \sqrt{\frac{3}{2}}$

e) $y = 8x - 8$ f) $y = \frac{4-x}{4}$.

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3. a) $f'(x) = -2x$ b) $f'(x) = 6x - 1$.

c) $f'(x) = 2x + 2$ d) $f'(x) = 4x^3$.

e) $f'(x) = \frac{3}{2\sqrt{x}}$ f) $f'(x) = \frac{1}{4}x + 1$.

4. a) $f'(x) = \frac{2}{3}$ b) $f'(x) = -7x^{-8}$.

c) $f'(x) = 0$ d) $f'(x) = 8x^7$.

5. a) $y = \frac{2}{3}x - 7$ b) $y = -7x + 8$.

c) $y = 8$ d) $y = 8x - 7$.

6. Hornið er 36.87° .

7. a) $3 - 2x^{-3}$ b) $\frac{|x|}{x} - 8x^7$.

c) $-3x^{-4} - 3x^2$ d) $\frac{1}{2\sqrt{x}} - 3x^2 - x^{-2}$.

8. a) $\frac{9}{2}\sqrt{x} - \frac{2}{\sqrt{x}}$ b) $6x + 7x^{-2}$.

c) $-3|x|x^{-5}$ d) $2x - 3\sqrt{x} + 1$.

e) $\frac{1}{x|x|}$

9. a) $6x^2 + 18x + 3$ b) $\frac{5x^2 - 6x + 1}{2\sqrt{x}}$.

c) $3x|x|$.

10.a) Margliðan er: $2x^3 - 3x^2 + x$

11.a) $\frac{-5}{(x-3)^2}$ b) $\frac{11}{(2x+3)^2}$.

c) $\frac{-6x^2 + 18x + 10}{(3x^2 + 5)^2}$ d) $-(x^{-2} + 8x^{-3})$

e) $-\frac{x^2 + 1}{(x^2 - 1)^2}$ f) $\frac{2x}{(1 - x^2)^2}$.

12.a) $\frac{-25}{(5x-3)^2}$ b) $\frac{-13}{(4x-1)^2}$

c) $\frac{-2x^2 + 18x + 10}{(3x^2 + 5)^2}$ d) $\frac{-2x}{(x^2 + 5)^2}$

e) $\frac{4x}{(1+x^2)^2}$.

f) $\frac{-x^2 + 2x}{(x^2 - x + 1)^2}$

13.a) $\frac{14-4x}{(x+2)^3}$ b) $\frac{x(2-x)}{(x^2-x+1)^2}$

c) $2x$ d) $\frac{x^4 - 2x^3 + 3x^2 - 4x + 2}{(x^2 - x + 1)^2}$

14.a) $4x^3 + 5$

b) $\frac{2x^3 + 8x^2 + 8x + 8}{(x+2)^2}$ c) 1

15.a) $80x^3 - \frac{12}{x^3}$ og $240x^2 + \frac{36}{x^4}$.

b) $\frac{-8}{(x+3)^3}$ og $\frac{24}{(x+3)^4}$.

c) $3x|x|$ og $6|x|$.

d) $48x - 24$ og 48.

16.a) $6(2x-7)^2$ b) $-6(2x-7)^2$.

c) $\frac{-6}{(3x-5)^3}$

d) $5\left(x^2 - \frac{1}{x}\right)^4 \left(2x + \frac{1}{x^2}\right)$.

17.a) $-2\left(x + (3x-1)^3\right)^{-3} \left(1 + 9(3x-1)^2\right)$

b) $\frac{3x^5 - 4x^{-9}}{\sqrt{x^6 + x^{-8}}}$

c) $3\left(2x + \sqrt{x}\right)^2 \frac{1 + 4\sqrt{x}}{2\sqrt{x}}$

18) Liðir a) og b). eru fyrir mistök eins og í dæmi nr. 17.

c) $5\left(x + \frac{1}{3x-1}\right)^4 \left(1 - \frac{3}{(3x-1)^2}\right)$

d) $\frac{1}{2\sqrt{\sqrt{x^4+1}+x}} \left(\frac{2x^3}{\sqrt{x^4+1}} + 1\right)$

19) Rúmmál kúlu með radíusinn r er

$$V = f(r) = \frac{4\pi r^3}{3}. \text{ En } r \text{ er fall af}$$

tíma, t , þ.e. $r = g(t)$, og $g'(t) = 1$. Þá er

$$V = \frac{4\pi}{3} g(t)^3 \quad \text{og} \quad \frac{dV}{dt} = 4\pi g(t)^2 \cdot g'(t).$$

Nú er gefið að $g'(t) = 1 \frac{\text{cm}}{\text{sek}}$ og

$g(t) = 5 \text{ cm}$, þ.e.

$$\frac{dV}{dt} = 4\pi g(t)^2 \cdot g'(t) = 100\pi \frac{\text{cm}^3}{\text{sek}}$$

20. $g = f \circ f \Rightarrow g' = f' \circ f \cdot f'$, $f(0) = 0$

$$g'(0) = f'(f(0)) \cdot f'(0) = f'(0) \cdot f'(0) = 4$$

$$j = f \circ f \circ f = f \circ g \Rightarrow j' = f' \circ g \cdot g'$$

$$j'(0) = f'(g(0)) \cdot g'(0) = f'(0) \cdot g'(0) = 8$$

21.a) $3(x^{-3} + \sqrt{3x})^{-4} \left(3x^{-4} - \frac{\sqrt{3}}{2\sqrt{x}} \right)$

b) $\frac{3}{2x} \sqrt{\frac{5-|x|}{3x-4}} \frac{5x+2|x|(x-2)}{(5-|x|)^3}$

c) $\frac{\left(1 + \sqrt{2x + \sqrt{3x}}\right) \left(2 + \frac{\sqrt{3}}{2\sqrt{x}}\right)}{2\sqrt{x + \sqrt{2x + \sqrt{3x}}} \cdot \sqrt{2x + \sqrt{3x}}}$

d) Setjum $f(x) = (x + \sqrt{x})^{-1}$,

$$g(x) = (x + f(x))^{-2} \text{ og}$$

$$h(x) = (x + g(x))^{-3}, \text{ en spurt er um}$$

$h'(x)$. Diffnunin fer þannig fram að

$f'(x)$ er sett inn í $g'(x)$ og útkoman sett inn í $h'(x)$, en fyrst er reiknað:

$$f'(x) = \frac{-(1 + 2\sqrt{x})}{2\sqrt{x}(x + \sqrt{x})^2},$$

$$g'(x) = \frac{-2(1 + f'(x))}{(x + f(x))^3} \text{ og}$$

$$h'(x) = \frac{-3(1 + g'(x))}{(x + g(x))^4}$$

22. Í öllum liðum er $f'(x)$ síjákvætt

a) $(f^{-1})'(5) = \frac{1}{f'(2)} = \frac{1}{3}$

b) $(f^{-1})'(5) = \frac{1}{f'(-2)} = \frac{1}{-1} = -1$

c) $(f^{-1})'(5) = \frac{1}{f'(1)} = \frac{1}{11}$

23.a) $\cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$ b) $\frac{-1}{\cos^2(x)}$

c) $-\sin\left(\frac{2\pi}{3}x\right) \frac{2\pi}{3}$

d) $\frac{-3\sqrt{x}}{2\sin^2(x\sqrt{x})}$

24.a) $3\sin^2(x)\cos(x)$

b) $\frac{2\cos(\tan(2x-5))}{\cos^2(2x-5)}$

c) $\frac{1+2\tan(x)}{\cos^2(x)}$

d) $\frac{x+2}{2(x+1)\sqrt{x+1}} \cos\left(\frac{x}{\sqrt{x+1}}\right)$

25.a) $\frac{\sin\left(\frac{x}{2}\right)}{\cos^3\left(\frac{x}{2}\right)}$

b) $\sin(x)(\cos^2(x) + \cos(2x))$

c) $\frac{-4x\cos(x^2)}{\sin^3(x^2)}$

26.a) $\sin(2x)$

b) $-\left(x\cos\left(\frac{1}{x}\right)\right)^{-2}$

c) $\frac{x\sin(2x^2)}{\sqrt{1+\cos^2(x^2)}}$

10 Svör við verkefnum

$$27.a) \frac{15 \cdot x^{\frac{-5}{8}}}{8}, \quad b) \frac{-15 \cdot (5x)^{\frac{-11}{8}}}{8}$$

$$c) \frac{-2}{3} (\sqrt{x} + x^2)^{\frac{-5}{3}} \left(\frac{1}{2\sqrt{x}} + 2x \right)$$

$$d) \frac{4x^3}{3} (1 + x^4)^{\frac{-2}{3}}$$

$$e) \frac{1}{10\sqrt{x}} (1 + \sqrt{x})^{\frac{-4}{5}}$$

$$28.a) \frac{-2}{5} \cos^{\frac{-3}{5}}(x) \sin(x)$$

$$b) \frac{-2x^{\frac{-5}{3}}}{3} \sin\left(x^{\frac{2}{3}}\right)$$

$$c) \sqrt{x} \frac{2x \cos(x) - 3(1 + \sin(x))}{2x^3}$$

$$d) D_f = \{x \mid \tan(x^2 + 1) > 0\} \text{ þar sem}$$

veldisvísirinn, þ.e. $\frac{-1}{3}$ er brot. Því má

sleppa algildismerkinu og svarið er:

$$\frac{-8x \tan^{\frac{-19}{15}}(x^2 + 1)}{15 \cos^2(x^2 + 1)}$$

$$e) \frac{(2x - |\sin(x)| \cot(x))}{6(x^2 + |\sin(x)|)^{\frac{5}{6}}}$$

$$29.a) \frac{2}{7} \left(\frac{x^3 + 1}{x^2} \right)^{\frac{-5}{7}} (1 - 2x^{-3})$$

$$b) \frac{-7}{10} \left(\frac{(x^{-1} - 1)}{(x + 1)} + |2x| \right)^{-1.7} \cdot \left(\frac{x^2 - 2x - 1}{x^2(x + 1)^2} + 2 \frac{|x|}{x} \right)$$

$$c) \frac{-3 |x^3 - x|^{\frac{-3}{2}} (3x^2 - 1)}{2 x^3 - x}$$

$$30.a) 9e^{3x} \quad b) -9e^{-3x}$$

$$c) -7e^{-x}(x - 1) \quad d) -2xe^{-x^2}$$

$$e) e^x (\cos(2x) - 2 \sin(2x))$$

$$f) e^{\tan(x)} (1 + \tan^2(x))$$

$$g) e^x \cos(e^x)$$

$$h) \frac{e^{2x} (2 \sin(x) - \cos(x))}{\sin^2(x)}$$

$$31.a) 5^{2x} \ln(25) \quad b) -5^{1-x} \ln(5)$$

$$c) \frac{2^{\tan(x)} \ln(2)}{\cos^2(x)} \quad d) 0$$

$$e) -\ln(3) \sin(x) \sin(2 \cdot 3^{\cos(x)}) 3^{\cos(x)}$$

$$f) \frac{\ln(5) \sin(2\sqrt{x})}{2\sqrt{x}} 5^{\sin^2(\sqrt{x})}$$

$$g) \pi^{x+e^x} \quad h) \frac{\ln(2)}{x} 2^{\ln(x)}$$

$$i) \frac{2 \ln(x)}{x}$$

$$32.a) 1 \quad b) \ln(2) \quad c) \cot(x)$$

$$d) \frac{3^{-x}}{x} - \ln(3) 3^{-x} \ln(x)$$

$$e) 4x^3 \ln(x) \quad f) 3 \ln(ex)$$

$$g) -3 \ln^2(\cos(x)) \tan(x)$$

$$h) \frac{2}{\ln(10)} \quad i) \frac{1}{\ln(2)}$$

$$33.a) ex^{e-1} \quad b) \frac{\pi}{2} (\sqrt{x})^{\pi-2}$$

$$c) \frac{(e \cot(x) - \ln(3)) \cdot \sin^e(x)}{3^x}$$

$$d) \frac{\pi}{x} \ln^{\pi-1}(x)$$

$$e) \frac{\sqrt{2}x^{\sqrt{2}-1} + 8^x \ln(8)}{\ln(8)(x^{\sqrt{2}} + 8^x)}$$

34.a) $1 + \ln(x)$

b) $x^{\ln(x)-1} \ln(x^2)$

c) $2(\sin^{2x}(x))(\ln(\sin(x)) + x \cot(x))$

d) $\frac{x^{\sqrt{x}} \ln(e^2 x)}{2\sqrt{x}}$

e) $\ln^{\ln(x)}(x) \frac{\ln(e \ln(x))}{x}$

f) $x^{\sin(x)} \left(\ln(x) \cos(x) + \frac{\sin(x)}{x} \right)$

g) $(x^{-2} + 4)^x \cdot \left(\ln\left(\frac{1}{x^2} + 4\right) - \frac{2}{4x^2 + 1} \right)$

h) $x^{2x} 2^x \left(\frac{1}{x} + \ln\left(2^{\ln(x)}\right) \right)$

Verkefni 5

1. a) $(1, -3)$ og $\left[\frac{1}{9}, \frac{-473}{243} \right] \cdot \left] -\infty, \frac{1}{9} \right]$,

$\left[\frac{1}{9}, 1 \right]$ og $[1, \infty[$.

b) $(-1, 2)$ og $(1, -2)$. $] -\infty, -1]$, $[-1, 1]$ og $[1, \infty[$.

c) $(1, 2)$. Bilin eru $[0, 1]$ og $[1, \infty[$.

d) $(0, 0)$. Bilin eru $] -\infty, 0]$ og $[0, \infty[$.

2. a) Vaxandi. b) Vaxandi.

c) Hvorugt. d) Hvorugt

3. a) Lotan er 2π . Hágildi: 0.369 í $x = 1.003$ og 1.7602 í $x = 3.776$. Lággi: -0.369 í $x = 2.139$ og -1.7602 í $x = 5.648$ (radíanmál).

b) Lotan er 2π . Hágildi: 1.125 í $x = 6.0305$ og $x = 3.3943$. Lággi: -2 í $x = \frac{\pi}{2}$ og 0 í $x = \frac{3\pi}{2}$.

c) Lotan er 2π . Hágildi: 1 í $x = \frac{3\pi}{2}$ og 0.2722 í $x = 0.4205$ og $x = 2.7211$.

Lággi: -1 í $x = \frac{\pi}{2}$ og -0.2722 í $x = 3.5621$ og $x = 5.8627$.

d) Lotan er π . Hágildi: 5 í $x = 0.4636$. Lággi: -5 í $x = 2.0344$.

e) Lotan er π . Hágildi: 4.472 í $x = 2.5880$. Lággi: -4.472 í $x = 1.0172$.

4. a) $D_f = \mathbb{R}$. Lággi: 0 í $x = -1$.

b) $D_f = \mathbb{R}$. Lággi: $2e^{-2}$ í $x = 2$.

c) $D_f = \mathbb{R}$. Hágildi: 0.2545 í $x = 1 + \sqrt{5}$. Lággi: -8.5092 í $x = 1 - \sqrt{5}$.

d) $D_f = \mathbb{R}$. Lággi er $-\frac{9}{4}$ í $x = \frac{\ln\left(\frac{3}{2}\right)}{\ln(2)}$.

5. a) Útgildin eru 17 og 12 . $f(1) = 17$ og $f(2) = 12$

b) Útgildin eru ± 2 , $f(-\sqrt{2}) = -2$ og $f(\sqrt{2}) = 2$.

c) Útgildin eru $\frac{1}{2}$ og $\frac{-1}{6}$, $f(-1) = \frac{1}{2}$ og $f(3) = \frac{-1}{6}$

d) Útgildin eru 10 og -22 , $f(-1) = 10$ og $f(3) = -22$.

6. a) $\left(\frac{5}{9}, \frac{-601}{243} \right)$

b) $(0, 0)$, $\left(\frac{-1}{\sqrt{2}}, \frac{7}{4\sqrt{2}} \right)$, $\left(\frac{1}{\sqrt{2}}, \frac{-7}{4\sqrt{2}} \right)$.

c) Engin beygjuskil.

d) Engin beygjuskil.

7. a) $(0, 0)$, $(0.12533, 0)$, $(3.01626, 0)$ og $\left(\frac{3\pi}{2}, 0 \right)$.

12 Svör við verkefnum

- b) $(0.704, -0.4855), (2.4375, -0.4855),$
 $(4.0242, 0.5793), (5.40063, 0.5793),$
- c) $(1.0157, -0.3777), (2.1259, -0.3777),$
 $(4.1573, 0.3777), (5.2675, 0.37771).$
- d) $(1.24905, 0), (2.81984, 0),$
- e) $(0.321751, 0), (1.89255, 0).$
8. a) $(0, \ln(2)), (-2, \ln(2)).$
 b) $(2, 2e^{-2}).$
 c) $(2 - \sqrt{6}, -0.44949),$
 $(2 + \sqrt{6}, 4.44949)$
 d) $(-0.41504, -1.6875).$
9. a) Sker x -ás í -1.5874 og 0 . Lágildi í
 $\left(-1, \frac{-3}{2}\right)$. Engin beygjuskil.
- b) Sker x -ás í -0.892122 og -0.486413 .
 Hágildi í $\left(1, \frac{5}{3}\right)$. Lágildi í
 $\left(\frac{-1}{\sqrt{2}}, \frac{9-8\sqrt{2}}{12}\right)$ og $\left(\frac{1}{\sqrt{2}}, \frac{9+8\sqrt{2}}{12}\right)$.
 Beygjuskil í $x = \frac{2 \pm \sqrt{10}}{6}$.
- c) Sker x -ás í 1 og $\frac{-1}{3}$. Lágildi í $(0, -1)$.
 Beygjuskil í $(1, 0)$ og $\left(\frac{1}{3}, \frac{-16}{9}\right)$. x -ás er
 snertill í $(1, 0)$.
- d) Sker x -ás í $x = 2 \pm \sqrt{2}$. Lóðfella er $x =$
 4 og skáfella $y = x$. Engin beygjuskil.
 Lágildi er í $(4 + \sqrt{2}, 4 + 2\sqrt{2})$, en
 hágildi í $(4 - \sqrt{2}, 4 - 2\sqrt{2})$.
- e) Sker x -ás í $\frac{1}{2}$ og 2 . Lóðfella: y -ás.
 Láfella: x -ás. Hágildisp.
 $\left(\frac{5+\sqrt{13}}{2}, 0.219855\right)$. Lágildisp. er
 $\left(\frac{5-\sqrt{13}}{2}, -1.51615\right)$. Beygjuskil í
- $(.910546, -1.18494),$ og
 $(6.58945, 0.195354).$
- f) Ferillinn sker x -ás x -ásinn í 0 . Lágildi
 er í $(0, 0)$. Beygjuskil eru í $(-\sqrt{6}, 6)$
 og $(\sqrt{6}, 6)$.
10. a) Lotan er 2π . Ferillinn sker x -ásinn í
 $\frac{\pi}{6}, \frac{5\pi}{6}$ og $\frac{3\pi}{2}$. Lágildispunktur:
 $\left(\frac{\pi}{2}, -2\right)$ og $\left(\frac{3\pi}{2}, 0\right)$. Hágildið: $\frac{9}{8}$ er
 fyrir $x \in \{3.39427, 6.03051\}$.
 Beygjuskil eru í punktum
 $(0.70412, -0.485523),$
 $(2.43747, -0.485523),$
 $(4.02415, 0.579273)$ og
 $(5.40063, 0.579273).$
- b) Lotan er π . Ferillinn sker x -ás í
 0.32175 og 1.89255 . Hágildið 5 er í
 2.67795 . Lágildið, -5 , er í 1.10715 .
 Beygjuskil eru í skurðpunktum við
 x -ásinn.
- c) Lotan er π . Ferillinn sker x -ás í
 1.10715 og 2.67795 . Hágildið 5 er í
 $x = 0.321751$. Lágildið -5 er í
 $x = 1.89255$. Beygjuskil eru í
 skurðpunktum við x -ásinn.
- d) Lotan er π . Ferillinn sker x -ásinn í
 0.463648 og 2.03444 . Hágildið 5 er í
 $x = 2.81984$. Lágildið -5 er í
 $x = 1.24905$. Beygjuskil eru í
 skurðpunktum við x -ásinn.
- e) Ferillinn sker x -ás í $\frac{\pi}{2}$ og $\frac{3\pi}{2}$.
 Hágildið $\frac{3\sqrt{3}}{2}$ er í $x = \frac{11\pi}{6}$.
 Lágildið $\frac{-3\sqrt{3}}{2}$ er í $x = \frac{7\pi}{6}$.

- Beygjuskil eru í skurðpunktunum við x -ásinn.
- f) $D_f = \mathbb{R}$. Ferillinn sker ekki x -ásinn.
 Hággildið 5.17387 er í $x = -\sqrt{2}$.
 Lágildið 0.592472 er í $x = \sqrt{2}$.
 Beygjuskil eru fyrir $x = -1 \pm \sqrt{3}$ í punktunum $(-2.73205, 4.84144)$ og $(0.73205, 2.06995)$. x -ás er láfella.
- g) $D_f = \mathbb{R}$. Ferillinn sker x -ás í -1 og 3 .
 Hággildið 0.691728 er í $x = -\sqrt{5}$.
 Lágildið -23.1305 er í $x = \sqrt{5}$.
 Beygjuskil eru fyrir $x = -1 \pm \sqrt{6}$ í punktunum $(-3.44949, 0.501772)$ og $(1.44949, -16.1829)$. Láfella: x -ás.
- h) $D_f = \mathbb{R}$. Ferillinn sker x -ás í 0 og $\frac{\ln(3)}{\ln(2)}$. Lágildið er -1 í $x = 1$.
 Beygjuskil eru í punktinum $(0,0)$.
 láfella er $y = 3$. $V_f = [-1, \infty[$
- i) $D_f = \mathbb{R}$. Ferillinn sker x -ás í 0 .
 Beygjuskil eru í punktinum $(0,0)$.
 $V_f = \mathbb{R}$.
- 11 4.
- 12 Rúmmálið verður stærst 128 dm^3 þegar afskornu ferningarnir eru $2 \times 2 \text{ dm}^2$.
- 13 6.25.
- 14 a) $a = -3$ og $b = -9$
 b) $a = -3$ og $b = -24$
- 15 3.9665° C .
- 16 1152.
- 17 $864\pi = 2714.34$.
- 18 $\frac{3}{2}$.
- 19 a) Staður 45.32 m. Hraði 11.436 ms^{-1} .
 Hröðun -9.8 ms^{-2} .
 b) Mesta hæð 52.37 m.
 c) Lokahraðinn er -32.364 ms^{-1} .
 (Hraðinn er að sjálfsögðu -31.36 ms^{-1} , þegar hluturinn er í hæðinni 2.2 m á niðurleið en hér er gert ráð fyrir að jörð tákni hæðina 0.)