

Svör

Svör við verkefnum

Verkefni 1

1. a) $\frac{\sqrt{10}}{3}$ b) $\frac{3\sqrt{2}}{4}$ c) ± 3

2. a) $\frac{13}{14}$ b) $\frac{14}{13}$

4. a) $\frac{15}{8}$ b) $\frac{1}{3}$

6. a) $\frac{1}{2}(\cosh(7x) + \cosh(3x))$

b) $\frac{1}{2}(\sinh(7x) + \sinh(3x))$

c) $\frac{1}{2}(\sinh(7x) + \sinh(3x))$

7. a) $2\cosh\left(\frac{7x}{2}\right)\cosh\left(\frac{3x}{2}\right)$

b) $2\sinh\left(\frac{9x}{2}\right)\cosh\left(\frac{x}{2}\right)$

c) $2\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{9x}{2}\right)$

8. a) $e^x = \sqrt[4]{\sqrt{5} - 2}$

b) $e^x = \sqrt[4]{5 \pm 2\sqrt{6}}$

c) $e^x = \sqrt[4]{2}$

9. a) $3\sinh^2(x)\cosh(x)$

b) $-x^{-2}\cosh\left(\frac{1}{x}\right)$

c) $\frac{3}{2}x^{\frac{1}{2}}\left(1 - \tanh^2\left(x^{\frac{3}{2}}\right)\right)$

d) $2\cosh(2x)$

10.a) $\sinh(2x)\left(1 + 4\sinh^2(x)\right)$

b) $2\sinh(2x)\cosh(x)\left(1 + 3\sinh^2(x)\right)$

c) $\sinh(2x)\left(1 + 6\cosh^2(2x)\right)$

d) $\frac{\cosh(x)\cosh(2x) + \sinh(x)\sinh(2x)}{\sqrt{\cosh(2x)}}$

11.a) $4\left(1 - \tanh^2(4x)\right)$

b) $\frac{5\sinh(5x)}{2\sqrt{2\cosh(5x) + 2}}$

12.a) $\frac{\sinh(x)}{2\cosh^2(x)} + \arctan\left(e^x\right)$

b) $\frac{1}{4}x^2 - \frac{1}{2}\ln(x)$

c) $\frac{1}{15}\sinh(x)\left(3\cosh^4(x) + 4\cosh^2(x) + 8\right)$

d) $-\frac{1}{2}\left(\tanh^2(x) + \ln\left(\tanh^2(x) - 1\right)\right)$

13.a) 4.26905

b) 0.321095

c) 0.179570

d) 4.43823

Verkefni 2

1. a) 48.7994

b) 39.1652

c) 20.4204

d) 52.8835

2. a) 227.870

b) 174.254

c) 201.062

d) 70.6858

3. a) 107.712

b) 49.2364

c) 10.0531

d) 90.4779

4. a) 1.23370

b) 0.693891

c) 2.98718

d) 0.519779

e) 1.14235

f) 2.44737

5. a) 1.85905

b) 0.549304

6. a) 3.54317

b) 9.28239

c) 9.45252

d) 2.36619

e) 6.75300

f) 9.28239

7. a) 268.083

b) 1392.998

8. 113.489

9. 1.98587

10. $k = 100.5$ eða $k = -99.5$

11. $V_1 = 5.23599$ og $V_2 = 28.2743$

2 Svör við verkefnum

$$12. d = \sqrt{\frac{20}{p}}$$

- 13.a) 94.7815 b) 12.5664
 c) 142.172 d) 18.9563
 e) 67.0206 f) 47.7809
 14.a) 31.4159 b) 57.6841
 c) 87.9646 d) 43.4146
 15.a) 6.70206 b) 26.8083
 c) 60.3186 d) 60.3186
 16.a) 49.4801 b) 113.097
 c) 189.924 d) 104.174
 17.a) 4.79992 b) 4.18879
 c) 7.76645
 18.a) 1.87636 b) 11.7692
 c) 9.53105
 19.a) 3.90197 b) 2.25655
 c) 20.9814 d) 10.3098
 20.a) 0.104720 b) 0.418879
 c) 2.19911 d) 1.67552
 21.a) 0.140859 b) 0.457168
 22.a) 53.4071 b) 118.124
 c) 80.0665 d) 87.9880
 23 a) 0.935085 b) 2.67307
 24.a) 49100.4 b) 5.12283

Botn (endaflétir) eru ekki meðtaldir!

Verkefni 3

1. a) $f((0,0)) = 3\sqrt{5}$, $f((-11, -2)) = \sqrt{314}$
 b) $V_f = \mathbb{R} \setminus \mathbb{R}_-$
 c) (0,11) og (0,-5)
 d) $f((x,y)) = \sqrt{(x-6)^2 + (y-3)^2}$
 e) $f(M) = [3\sqrt{2.5}, \infty[$
2. a) $f((0,0)) = 0$, $f((-3,4)) = 5$
 b) $V_f = \mathbb{R} \setminus \mathbb{R}_-$
 c) (0,10) og (0,-10)
 d) $f((x,y)) = \sqrt{x^2 + y^2}$
3. a) (0,0) b) $x + y = 0$
 c) $(x+2)^2 + (y-1)^2 = 1$
 d) $(x+2)^2 + y = 2$

$$4. a) M = \{(x,y) \mid y = x\}$$

$$b) M = \{(x,y) \mid (x+1)^2 + (y+1)^2 = 1\}$$

5. a) (-3,-1) b) (5,1)
 6. a) $y = x + 2$ b) $y = x - 4$
 7. $k_y = \sqrt[3]{0.25}$
 8. a) $f((0,0)) = (-3,0)$, $f((-1,3)) = (-1,3)$
 b) $f((x,y)) = (-2x - 3, y)$
 c) $3x - 8y - 15 = 0$
 9. $k = \pm\sqrt{3}$

$$10. a) y = \left(x - \frac{1}{2}\right)^2 - 4, y = \frac{2}{2x-1} - 4,$$

$$y = \sqrt{x - \frac{1}{2}} - 4, y = \frac{2}{x} - 4$$

$$b) y = -3\left(x - \frac{1}{2}\right)^2 - 4, y = \frac{-6}{2x-1} - 4,$$

$$y = -3\sqrt{x - \frac{1}{2}} - 4, y = \frac{-6}{x} - 4$$

$$c) y = -3\left(x - \frac{1}{2}\right)^2 + 12, y = \frac{-6}{2x-1} + 12,$$

$$y = -3\sqrt{x - \frac{1}{2}} + 12, y = \frac{-6}{x} + 12$$

11. a) (-2,-3) b) (13,17)
 12. a) (1,0) b) $(x+4)^2 + y^2 = 4$
 13. a) $\left(\frac{\sqrt{3}-1}{2}, \frac{-\sqrt{3}-1}{2}\right)$ b) $\left(\sqrt{3} + \frac{1}{2}, 1 - \frac{\sqrt{3}}{2}\right)$
 c) $\left(\frac{1-\sqrt{3}}{2}, \frac{\sqrt{3}+1}{2}\right)$ d) $\left(\frac{\sqrt{3}-1}{2}, \frac{-\sqrt{3}-1}{2}\right)$
 14. $(\sqrt{3} + 3, \sqrt{3} + 1)$

Verkefni 4

1. a) $(2\sqrt{5})_{\arctan(2)}$ b) $(2\sqrt{5})_{\arctan(0.5)}$
 c) $(2\sqrt{5})_{\arctan(-2)}$ d) $(2\sqrt{5})_{p - \arctan(0.5)}$
 e) 4_p f) $(\sqrt{29})_{p - \arctan(\frac{2}{3})}$
2. a) (2.16121, -3.26588)

b) $(0,2)$ c) $(-p,0)$

d) $(\sqrt{2}, -\sqrt{2})$

e) $(-1.97998, -0.28224)$

f) $(-1.24844, -2.72789)$

3. a) $x^2 + y^2 = 25^2$ b) $y = 0 \wedge x < 0$

c) $y = -\sqrt{3} \cdot x$ d) $\sqrt{3} \cdot x = |y|$

4. a) $r(3\cos(q) + 4\sin(q)) = 5$

b) $r(3\cos(q) - 4\sin(q)) = 5$

c) $r(3\cos(q) - 4\sin(q)) = -5$

d) $r(5\cos(q) - 12\sin(q)) = 13$

5. a) $r = \frac{2\sqrt{6}}{\sqrt{1+7\cos(2q)}}$

b) $r(\sin(q) + \cos(q) - r^2 \sin^3(q)) = 2$

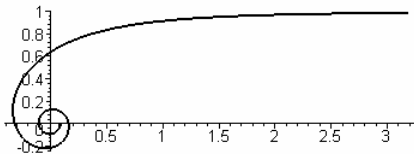
6. a) $0 < x^2 + y^2 < 9$ b) $x^2 + y^2 > 16$

7. a) $x = 0 \wedge y \neq 0$ b) $y = -\sqrt{3} \cdot x$

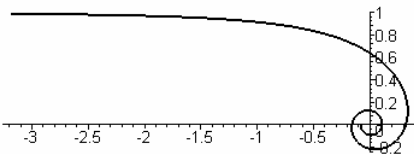
8. a) Hallahorn 90° b) Hallahorn 0°

c) 1.47663

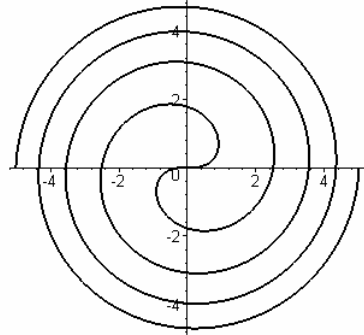
9. a) $a = 2 \log 0.3 \leq q \leq 4p$



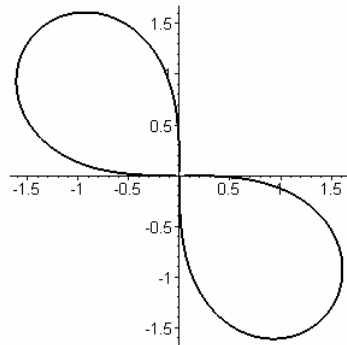
$a = 2 \log -4p \leq q \leq -0.3$



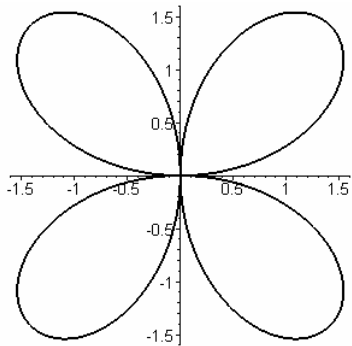
b) $a = 2 \log 0 \leq q \leq 4p$



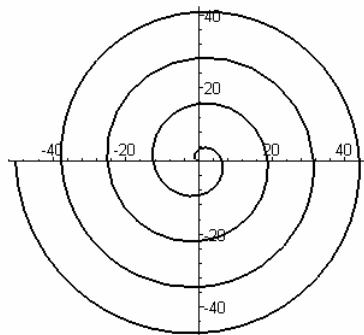
c)



d)



e)



4 Svör við verkefnum

10. a) 4.83763 b) 1.36971
 11. a) 4.44063 b) 0.271758
 12. $\int_{n \cdot 2p}^{(n+1)2p} t^2 dt - \int_{(n-1)2p}^{n \cdot 2p} t^2 dt = 16 \cdot n \cdot p^3$
 $\int_{2p}^{4p} t^2 dt - \int_0^{2p} t^2 dt = 16 \cdot p^3$
 Viðaukinn á umferð nr. n er því $(n-1)$ -faldur viðaukinn á 2. umferð.
 13. a) 8 b) 8 c) 8
 14. a) 25π b) $(2-\sqrt{2})p$ c) $2\sqrt{2} \cdot p$

Verkefni 5

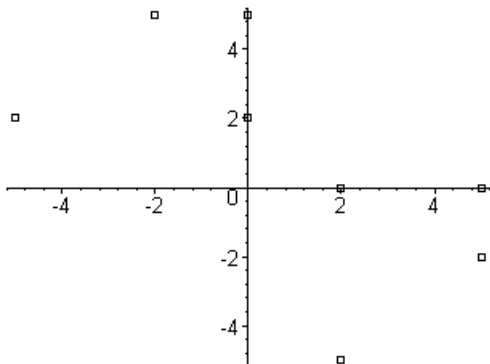
1. Ef e og f eru tvær samlagningarhlutleysur í \mathbb{C} , þá er $e + f = e$ því f er samlagningarhlutleysa og $e + f = f$ því e er samlagningarhlutleysa.

Af þessu sést að $e = f$.

Ef z_1 og z_2 eru saml.andhverfur z fæst $(z_1 + z) + z_2 = z_2$ því z_1 er saml.andhv. z .
 $z_1 + (z + z_2) = z_1$ því z_2 er saml.andhv. z .
 Þar sem tengireglan gildir um samlagningu sést að $z_1 = z_2$.

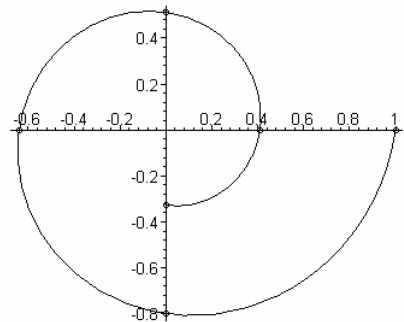
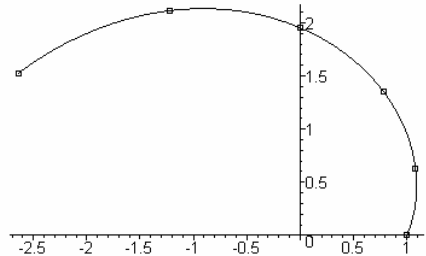
2. a) $-1 + i$ b) $-4 + 7i$
 c) $-\frac{8}{13} - \frac{1}{13}i$ d) $-117 - 44i$
 3. a) $\frac{16}{5} - \frac{2}{5}i$ b) $2 - 24i$
 c) -1 d) $\frac{115}{89} + \frac{172}{89}i$
 e) $3i$

4.



- a) $-2 + 5i$ b) 2 c) $5i$

- d) $5 - 2i$ e) $2i$ f) $-5 + 2i$
 g) 5 h) $2 - 5i$
 5. a) $\frac{2}{5} + \frac{9}{5}i$ b) $-\frac{1}{3}i$
 c) $z = 5, w = -2i$
 d) $z = \frac{139}{226} + \frac{175}{226}i, w = \frac{136}{113} - \frac{6}{113}i$
 6.



7. a) $256\sqrt{2} + 256\sqrt{2}i = 512 \frac{p}{4}$
 b) $-\frac{81}{2} - \frac{81\sqrt{3}}{2}i = 81 \frac{2p}{3}$
 c) $-1 = 1_p$
 d) $\frac{1}{2} + \frac{\sqrt{3}}{2}i = 1 \frac{p}{3}$
 e) $-\frac{7}{2} - \frac{7\sqrt{3}}{2}i = 7 \frac{2p}{3}$
 8. a) $512i = 512 \frac{p}{2}$
 b) $-\sqrt{2} - \sqrt{2}i = 2 \frac{3p}{4}$
 c) $1 = 1_0$ d) $-32 = 32_p$

$$9. a) z = \begin{cases} 5_{\frac{p}{3}} = \frac{5+i\sqrt{3}}{2} \\ -5 \\ 5_{\frac{5p}{3}} = \frac{5-i\sqrt{3}}{2} \end{cases}$$

$$b) z = \begin{cases} \left(\frac{1}{2}\right)_{\frac{p}{6}} = \frac{\sqrt{3}+i}{4} \\ \left(\frac{1}{2}\right)_{\frac{5p}{6}} = \frac{-\sqrt{3}+i}{4} \\ \left(\frac{1}{2}\right)_{\frac{3p}{2}} = \frac{-i}{2} \end{cases}$$

$$c) z = \begin{cases} 1.03359 + 1.28301i \\ 0.594319 - 1.53662i \\ -1.62791 + 0.253615i \end{cases}$$

$$d) z = 3 \cdot \text{cis}\left(\frac{p}{12} + j \cdot \frac{2p}{3}\right), j \in \{0, 1, 2\}$$

$$10.a) z = \pm(3 - i) \quad b) z = 0 \vee z = \pm 5i$$

$$c) z = \pm\sqrt{2} \cdot (1 + i)$$

$$d) z = 2 \cdot \text{cis}\left(\frac{-p}{8} + h \cdot \frac{p}{2}\right), h \in \{0, 1, 2, 3\}$$

$$11.a) z = \frac{-1 \pm i \cdot \sqrt{19}}{2}$$

$$b) z = \begin{cases} \frac{1}{2\sqrt{2}}(\sqrt{\sqrt{17}+1} + i\sqrt{\sqrt{17}-1}) \\ \frac{-1}{2\sqrt{2}}(\sqrt{\sqrt{17}+1} + i\sqrt{\sqrt{17}-1} - \sqrt{2}) \end{cases}$$

$$c) z = \begin{cases} \frac{1}{\sqrt{2}}(\sqrt{\sqrt{5}-1} - \sqrt{2} - i(\sqrt{\sqrt{5}+1} + \sqrt{2})) \\ \frac{-1}{\sqrt{2}}(\sqrt{\sqrt{5}-1} + \sqrt{2} + i(\sqrt{\sqrt{5}+1} + \sqrt{2})) \end{cases}$$

$$12.a) (z - 3i)(z + 2i)$$

$$b) (z - 2 - 3i)(z - 2 + 3i)$$

$$c) (z - i)(z + 1 - i)$$

$$d) (z + 3)(z + 5i)$$

$$13.a) -2, \text{ og } \pm 2i.$$

$$b) \pm 3, \text{ og } 1 \pm 3i.$$

$$c) \pm i \cdot \sqrt{2} \text{ og } \frac{-1 \pm i \cdot \sqrt{3}}{2}$$

$$d) \frac{-1}{2}, \frac{2}{3} \text{ og } 2 \pm i$$

$$14.a) -1 \pm i, \frac{-1}{3}$$

$$b) \pm \frac{1 \pm i}{2}, \text{ (allir fjórir möguleikar).}$$

$$c) \pm(1 + 2i).$$

$$d) \frac{1 - i \cdot (2 + \sqrt{3})}{2}, \frac{-1 + i \cdot (2 - \sqrt{3})}{2}$$

$$15.a) x = \ln(13), y = \arctan\left(\frac{-12}{5}\right)$$

$$b) x = \ln(3), y = \frac{p}{3}$$

$$c) x = \ln(5), y = \frac{3p}{2}$$

$$d) x = \ln(218), y = \arctan\left(\frac{13}{7}\right)$$

$$d) x = \ln(8), y = \frac{p}{4}$$

$$16.a) z = 2 + i \cdot \frac{1}{2}$$

$$b) z = \ln(10) - i \cdot \arctan\left(\frac{3}{4}\right)$$

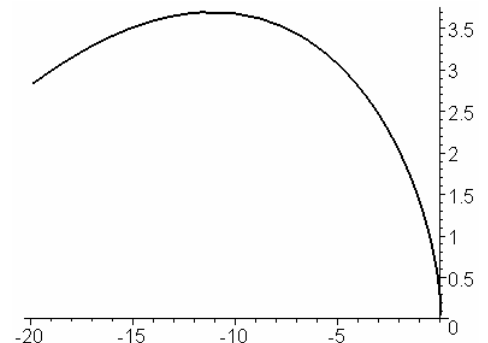
$$c) \frac{1}{2} \cdot \ln(2) + i \cdot \left(\frac{-p}{8} + h \cdot p\right), h \in \{0, 1\}$$

$$d) z = \ln(2) + i \cdot \left(\frac{p}{6} + h \cdot \frac{2p}{3}\right), h \in \{0, 1, 2\}$$

$$e) \text{Mod}(e^{i \cdot z}) = e^{-\frac{1}{2}}, \text{Arg}(e^{i \cdot z}) = \frac{\sqrt{3}}{2}$$

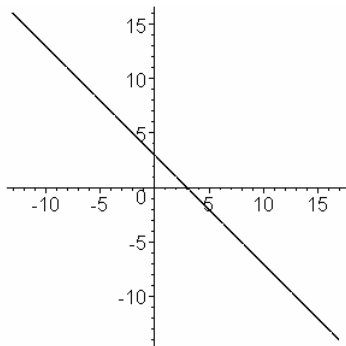
$$17.a) f'(t) = (3 + i)e^{2i+t(3+i)}$$

Á bilinu $[-1, 1]$ er ferillinn svona:

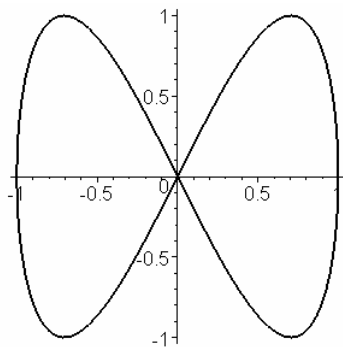


6 Svör við verkefnum

b) $f'(t) = 3 - 3i$



c) $f'(t) = \cos(t) - i \cdot \sin(2)$



18.a) $2t(e^{it} + i)(e^{it}(1 + i \cdot t) + i)$

b) $i \cdot e^{(1+i)t}(1 + (1+i)t)$

c) $\tan(t) \cdot (1 + i \cdot e^{-it}) + \frac{t - e^{-it}}{\cos^2(t)}$

Verkefni 6

1. a) $\frac{e^{2x}}{4} + c_1x + c_2$

b) $\frac{x^3}{2} + x^2 + c_1x + c_2$

c) $\frac{x^4}{6} - \frac{x^3}{2} + 2x^2 + c_1x + c_2$

d) $\frac{x^3}{6} + x(\ln(x) - 1) + c_1x + c_2$

2. a) $\sin(3x) + c_1x + c_2$

b) $\frac{1}{2}x^2 - \frac{1}{4}\cos(2x) +$

$\int \ln(\sin(x))dx + c_1x + c_2$

c) $e^{2x} + e^{-2x} + c_1x + c_2$

d) $c_1e^{-x} + c_2$

e) $c_1e^{-2x} + c_2$

3. a) $\frac{1}{2}x^2 - x + c_1e^{-x} + c_2$

b) $\frac{2}{3}x^2 + \frac{4}{9}x + c_1e^{3x} + c_2$

c) $\frac{1}{3}x^3 + c_1e^x + c_2$

d) $x^4 + c_1x^2 + c_2$

4. $y(t) = -\frac{g}{2}t^2 + 80 \cdot t + 2$, þ.e.a.s.

$y(t) = -4.9 \cdot t^2 + 80 \cdot t + 2$. En mesta hæð

er eftir $t_0 = \frac{80}{g} = 8.16$ sek. og er

$y_0 = \frac{8040}{g} + 2 = 328.2$ metrar.

5. a) $c_1e^{-5x} + c_2e^{-x}$

b) $c_1xe^{2x} + c_2e^{2x}$

c) $c_1xe^{5x} + c_2e^{5x}$

d) $e^{-\frac{x}{2}} \left(c_1 \cos\left(\frac{x\sqrt{3}}{2}\right) + c_2 \sin\left(\frac{x\sqrt{3}}{2}\right) \right)$

e) $e^x \left(c_1 \cos(x\sqrt{3}) + c_2 \sin(x\sqrt{3}) \right)$

6. a) $\frac{1}{2}e^{2x} - \frac{3}{2}e^{-x}$

b) $-\frac{1}{3}e^x - \frac{2}{3}e^{-\frac{x}{2}}$

c) $\frac{3}{4}(e^{2x} - e^{-2x})$

d) $8e^{-3x}$

e) $x \cdot e^{-x}$

7. a) $c_1 \cdot e^{2x} + c_2 \cdot e^x$

b) $c_1 \cdot e^{-5x} + c_2$

c) $c_1 \cdot e^{x(5-\sqrt{22})} + c_2 \cdot e^{x(5+\sqrt{22})}$

d) $c_1 \cdot \cos(5x) + c_2 \cdot \sin(5x)$

e) $e^{2x} \cdot (c_1 \cdot \cos(2x) + c_2 \cdot \sin(2x))$

f) $e^{8x} \cdot (x \cdot c_1 + c_2)$

8. a) $\frac{-x}{2} \cos(x) + c_1 \cos(x) + c_2 \cos(x)$

b) $e^{-x} \left(\frac{x^2}{2} + c_1x + c_2 \right)$

- c) $\frac{x}{2}e^x + c_1e^x + c_2e^{-x}$
 d) $e^{-2x}(c_1 \cos(x) + c_2 \sin(x)) + 2$
 e) $\frac{1}{8}\cos(2x) + e^{-2x}(c_1x + c_2)$
9. a) $\frac{1}{18}e^x(9e^x - 6x - 2 + c_1xe^{3x} + c_2)$
 b) $\frac{2}{5479}(73\cos(x) - 10\sin(x)) + e^{5x}(c_1 \cos(7x) + c_2 \sin(7x))$
 c) $\frac{3}{7}e^x + \frac{1}{13}(6\sin(x) - 4\cos(x)) + e^{-x}(c_1 \cos(x\sqrt{3}) + c_2 \sin(x\sqrt{3}))$
 d) $e^{2x}(c_1 \cos(2x) + (\frac{x}{2} + c_2)\sin(2x))$
 e) $\frac{1}{100} \cdot (16\cos(x) + 12\sin(x) + 75) + \frac{1}{2}x^2 + x - \frac{1}{3}e^x + e^{2x}(c_1x + c_2)$

10.a) $1 - x$

b) $e^{x(2+\sqrt{2})} + e^{x(2-\sqrt{2})}$

c) $e^{-2x} - 3 \cdot x \cdot e^{-x}$

11.a) $k_1 \cos(x) + k_2 \sin(x) - \frac{1}{2}\cos(x) - \frac{1}{2}x \sin(x) + k_3$

b) $\frac{1}{4}e^{-2x} + a \cdot e^{x \cdot 2\sqrt{2}} + b \cdot e^{-x \cdot 2\sqrt{2}}$

c) $c_1e^x + c_2e^{-x} - 2x + c_3 \cos(x) + c_4 \sin(x)$

d) $e^{-x}(c_1x + c_2) + \frac{1}{4}\sin(x) + e^x(\frac{1}{8}x^2 + c_3x + c_4)$

12.a) $y''(t) + 16 \cdot y(t) = 0 \Rightarrow$

$y(t) = y_0 \cdot \cos(w \cdot (t - t_0)),$ en

b) $y''(t) + y'(t) + 16k \cdot y(t) = 0 \Rightarrow$

$y(t) = e^{\frac{-1}{2}t}(c_1 \cos(wt) + c_2 \sin(wt)) = c_0 \cos(w(t - t_0)),$ en $w = \sqrt{\frac{23}{2}}$

c) $y''(t) + y'(t) + 16k \cdot y(t) = \sin(2t) \Rightarrow$

$y(t) = e^{\frac{-1}{2}t}(c_1 \cos(wt) + c_2 \sin(wt)) = c_0 \cos(w(t - t_0)),$ en $w = \frac{3\sqrt{7}}{2}$

13. $m \cdot y''(t) + c \cdot y'(t) + m \cdot g = 0 \Rightarrow$

$y(t) = -\frac{mg}{c}t - c_1 \cdot e^{-\frac{c}{m}t} + c_2$ En af

$y'(0) = v_0 \wedge y(0) = 0$ leiðir

$y(t) = -\frac{mg}{c}t - m \frac{mg + v_0c}{c^2} \cdot e^{-\frac{c}{m}t} + m \frac{mg + v_0c}{c^2}$

14. $m \cdot y''(t) + k \cdot y(t) = 0 \Rightarrow$

$y = c_1 \cdot \cos\left(\sqrt{\frac{k}{m}}t\right) + c_2 \cdot \sin\left(\sqrt{\frac{k}{m}}t\right) = c_0 \cdot \sin\left(\sqrt{\frac{k}{m}}(t - t_0)\right)$

Nú er $m = 12$ kg, $k = 523.2$ Nm⁻¹,

$y(0) = 0.3$ m, $y'(0) = 1.8$ m·s⁻¹ og

$y = c_1 \cdot \cos(6.603t) + c_2 \cdot \sin(6.603t).$

Af $y(0) = c_1 = 0.3$ og

$y'(0) = 6.603 \cdot c_2 = 1.8$ fæst

$y = 0.3\cos(11.44t) + 0.273\sin(11.44t)$

15. $8 \cdot y''(t) + 4 \cdot y'(t) + 2 \cdot y(t) = 0 \Rightarrow$

$y = e^{-\frac{t}{4}}\left(c_1 \cos\left(\frac{\sqrt{3}}{4}t\right) + c_2 \sin\left(\frac{\sqrt{3}}{4}t\right)\right)$